Scaling Laws of the Energy Confinement Time in Stellarators without Renormalization Factors

by A.Murari[1*], E.Peluso[2*], J. Vega[3], J.M.García-Regaña[3], J.L.Velasco[3], G.Fuchert[4] and M.Gelfusa[2]

1) *Consorzio RFX (CNR, ENEA, INFN, Università di Padova, Acciaierie Venete SpA), Corso Stati Uniti 4, 35127 Padova, Italy.*
2) *Department of Industrial Engineering, University of Rome "Tor Vergata", via del Politecnico 1, Roma, Italy*
3) *Laboratorio Nacional de Fusión, CIEMAT. Av. Complutense 40. 28040 Madrid. Spain*
4) *Max-Planck-Institut für Plasmaphysik, Greifswald, Germany*

*These authors contributed equally to the paper.

Abstract

Identifying a unified scaling law for the energy confinement time in Stellarators has proved to be a quite challenging task, given the flexibility and complexity of the configuration. The most widely accepted model is a power law and contains a normalization factor for each individual device (and even for each sufficiently different magnetic configuration in a single machine). In the last decade, new and very powerful data analysis tools, based on Symbolic Regression (SR) via genetic programming (GP), have become quite consolidated and have provided very interesting results for the Tokamak configuration. Application of SR via GP to the largest available multimachine Stellarator database permits to relax the power law constraint as an alternative to the use of renormalization factors. This approach has allowed converging on very competitive global scaling laws, which present exponential and squashing terms but do not contain any renormalization coefficient. Moreover, the exploratory application of SR via GP has revealed that the two main types of magnetic topology, with and without shear, can be much better interpreted with two different models. The fact that these new scaling laws have been derived without recourse to any renormalization increases their interpretative value. On the other hand, the techniques developed emphasise the need for improving the statistical basis before drawing definitive conclusions and providing reliable extrapolations.

Keywords: Multimachine databases, Scaling laws, Symbolic Regression, Genetic Programming, Energy Confinement Time, Stellarators

Corresponding author: gelfusa@ing.uniroma2.it

1 The Stellarator configuration and multimachine databases

The research on Magnetic Confinement Nuclear Fusion (MCNF) in the world is presently focussed specifically on the investigation of two main configurations, the Tokamak and the Stellarator. In the Tokamak, the plasma current is an essential ingredient in the formation and sustainment of the configuration [1]. The Stellarator, on the other hand, can be considered current-free, since the small unavoidable currents (diamagnetic, bootstrap etc.) normally do not play a fundamental role, because the basic confinement is insured by the topology of the fields, imposed from outside by suitable coils [2]. Due to the technological complexities of the current-free devices, fewer machines have been built and therefore the properties of the Stellarator are less known than the Tokamak's. Even for the Tokamak though, theoretical models for the energy confinement time are not available, due to the intricacies of the transport mechanisms involved. Moreover, the range of relevant scales is such that also numerical simulations are prohibitively expensive in realistic geometries and conditions. These difficulties have motivated the building of multimachine databases (DBs), for the extraction of empirical scaling laws directly from the data. The main rationale behind this choice resides in the fact that individual devices typically can scan only a limited interval of the most relevant physical parameters and can be affected excessively by machine specific spurious correlations. Interpreting and extrapolating the results, for example for the planning of new experiments or the design of new devices, become therefore particularly delicate tasks. Consequently, inputs from different experiments are expected to increase the statistical basis of the investigations and improve the robustness of the conclusions. Indeed, the main scaling laws for the Tokamak confinement time, such as the IPB98(y,2), have been derived from the international database of the International Tokamak Programme Agreement (ITPA) [3, 4], which contains entries from all the major devices ever operated in the world.

In the last two decades, the results and understanding of the Stellarator configuration have improved significantly, to the point that now multimachine databases for the investigation of the energy confinement time are available [5]. On the other hand, given the high flexibility of the current-free devices, identifying a unified scaling law for this magnetic configuration is a quite challenging task. Indeed, a model of sufficient robustness and generality is still missing; the most widely accepted scaling law, the International Stellarator Scaling 2004 (ISS04), includes a so-called renormalization factor f_{ren} that has to be independently fitted for each device and sufficiently different magnetic configuration [5]. The idea behind the renormalization factor is that the standard 0D quantities may be not enough to capture all the complexities of the configuration. The renormalization factor is supposed to help in this respect. For instance, f_{ren} is smaller

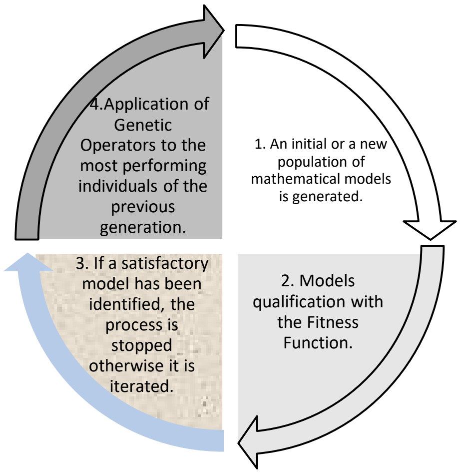

Figure 1. The Symbolic Regression via Genetic Programming approach to the derivation of scaling laws directly from the data.

for the so called "inward shifted" LHD configuration, which is known to exhibit reduced neoclassical losses, than for other configurations with larger neoclassical transport. The different values of f_{ren} are attributed to magnetic configuration features other than the minor radius *R*, minor radius *a*, the rotational transform iota $\iota_{2/3}$ and the global kinetic quantities (see Section 3).

The implementation of f_{ren}, of course, affects the interpretability and extrapolation reliability of the obtained models. First, f_{ren} tends to obfuscate the understanding of the physics and to render more delicate both the planning of new experiments and the comparison with the other configurations. Indeed, the renormalization factor can hide poorly understood correlations between important quantities; moreover there is no principled way to assess the confidence intervals of the equations containing these renormalization factors. Secondly, constraining the scalings to be in power law form does not help much, because power laws are too rigid to really reproduce the experimental data (Section 2 and 6). More sophisticated tools, such as Symbolic Regression (SR) via Genetic Programming (GP), have the potential to improve this unsatisfactory situation (Section 2). Their application to the Stellarator international database, which is overviewed in Section 3, is an alternative approach, which permits to increase the flexibility of the scalings mathematical form, without making recourse to renormalisation factors. The results of the analysis, for the largest internationally available public database, are reported extensively in Section 4, showing how the entries of the shearless configurations and of the ones with shear are better fitted with two different scaling laws. Conclusions and lines of future investigations are discussed in the last section of the paper.

2 Symbolic Regression via Genetic Programming and non-power law scalings for Stellarators

A common guiding principle, for extracting scalings from databases in thermonuclear fusion, has often been self-similarity, which allows justifying that the models are in power law monomial form. This assumption has been very convenient for a long time, since the only tool available to extract scaling laws from large databases was log regression. In the investigations of the Tokamak, the limitations of power law monomials have been documented since quite some time [6-13]. In the case of the Stellarator configuration, the rigidity of power laws is even more evident. Indeed, in the most advanced studies of Stellarators, the predictions of the models had to be multiplied by an "*ad hoc*" coefficient, called a renormalization factor in the literature, to be coherent with the experimental energy confinement time of the individual devices [5]. This evidence begs the question whether more general and more flexible mathematical expressions of the models would allow identifying unified scaling laws, without the need to introduce additional renormalization factors. Computational techniques to explore large databases for regression nowadays exist and the most advanced is probably Symbolic Regression via Genetic Programming [14, 15].

Symbolic Regression via Genetic Programming consists of a series of methodologies, which allow implementing a new approach to the extraction of scaling laws from large databases. The mathematical models for the scalings are derived directly from the data. This is achieved by manipulation of symbols, basic mathematical units, with genetic programming [14, 15]. Genetic Programs (GPs) work with a population of individuals, e.g. mathematical expressions, constituting candidate models of the scaling laws to be investigated. These models can be formulated as trees, which have a very good representational power and make easy the implementation of Genetic Programming. The algorithms of SR via GP traverse the database and test the quality of various formulas, generated as combinations of the initial basic units defined by the user. An overview of the entire methodology is provided in graphical form in Figure 1; for the details with applications to MCNF the reader is referred to [6-9].

One of the main elements of the procedure is the qualification of the various candidate formulas. This is achieved with a specific metric that, in this context, is called fitness function (FF). The selection of the best individuals, which are granted a higher probability to have descendants, is based on their score in terms of the FF. The genetic operators are applied to these best performing individuals to obtain the next generation. The process is iterated, until convergence to satisfactory solutions. The final output of the technique consists of a series of data driven models, whose mathematical form is particularly suited to interpreting the available database.

With regard to the FF, for the purpose of the present paper, the indicators used to determine the quality of the solutions are well-established model selection criteria. They belong to the classes of Bayesian and information theoretic metrics: the Bayesian Information Criterion (BIC), the Akaike Information Criterion (AIC) and the Takeuchi Information Criterion (TIC) are all consolidated and widely used indicators for this task [16]. Since all these metrics are conceptually similar, only the BIC is discussed in the following as a representative case. The BIC is an unbiased estimator of the likelihood of a model. The form of the BIC indicator used in this paper is:

$$BIC = n \cdot \ln\left(\sigma^2_{(\epsilon)}\right) + k \cdot \ln(n) \qquad (1)$$

where $\epsilon = y_{\text{data}} - y_{\text{model}}$ are the residuals, $\sigma^2_{(\epsilon)}$ their variance, k is the number of nodes in the tree and n the number of y_{data} available, so the number of entries in the database (DB).

BIC and the other criteria most commonly used are cost functions to be minimised, in the sense that better models have lower values of these metrics. The main ideas behind their formulation can be appreciated by inspection of the BIC mathematical structure. Indeed, BIC, as the other model selection criteria, consists basically of two parts. The first one depends on the quality of the fit, quantified by the residuals. Models closer to the data have lower values of this term. The second addend implements a penalisation for complexity, since it is proportional to the number of nodes in the tree representing the model equation. Therefore, parsimony is built in the cost function to avoid overfitting, contrary to the vast majority of the frequentist methodologies, which do not consider explicitly this issue and therefore do not include terms penalising excessive complexity of the models. All the mathematical background, to fully appreciate the relative merits of these model selection criteria, can be found in [16]. It is worth mentioning that, for the Stellarator database analysed and the models discussed in this paper, the three selection criteria (AIC, BIC and TIC) provide exactly the same classification, increasing the confidence in the derived results.

Very often, and this is certainly the case of the Stellarator database studied in the following, the quality and quantity of the entries, compared to the complexity of the scalings to be identified, is insufficient to converge on an unique best model. In this eventuality, a typical solution consists of making recourse to the Pareto Frontier (PF). The PF is the set of non-dominated optimal solutions, i.e. the set of best models, one for each level of complexity. The Pareto Frontier presents typically an L shape and the models around the inflexion point are the most likely candidates, because they constitute the best compromise between accuracy and complexity.

Another point to note, particularly relevant for the physical interpretation as described in the following sections, is the issue of estimating the confidence intervals. The models obtained with SR via GP do not contain any renormalization factor and therefore, with advanced but consolidated nonlinear fitting techniques [17], it is possible to calculate their confidence intervals. The non-linear procedure performed has already been described in [12] and therefore in the following only the main features are summarized. Non-linear fits [18] are performed to find the best parameters, exponents and constants, represented with the vector $\vec{c}$. This optimisation is achieved by minimizing the following functional, i.e. the sum of squares of the residuals $\vec{\varepsilon}$:

$$\min_{LB\le\vec{c}\le UB}\left[\sum\left(\vec{\varepsilon}(\vec{x};\vec{c},\vec{c}_0)\right)^2\right]=\min_{LB\le\vec{c}\le UB}\left[\sum\left(\vec{y}_{data}(\vec{x})-\vec{y}_{mod}(\vec{x};\vec{c},\vec{c}_0)\right)^2\right]$$

Where $\vec{x}$ stands for the quantities mined from the DB, considered as independent.

The risk of selecting a local solution has been minimized with the help of an iterative procedure. Consequently, at each iteration, each set $\vec{c}$ is perturbed and, if all the values assumed by the parameters do not change within a $1e^{-6}$ tolerance level between two consecutive iterations, the convergence has been reached. It is worth mentioning that such approach has been also used to minimize different functionals based on different metrics, such as the Geodesic distance [10,13].

The main steps of the methodology are then the following: first the 99% confidence intervals of the initial set of parameters $\vec{c}_0$ are evaluated with an initial iteration by a bootstrap procedure; these intervals are consequently used as upper bounds (UB) and the lower bounds (LB) of the parameters to be found. Then the routine, using a trust region method based on an interior reflective Newton algorithm [19], converge on the final set of parameters $\vec{c}$ after a certain amount of iterations. Finally the 95% confidence intervals are evaluated for the final set of parameters found. It is then possible to estimate the statistical criteria reported in the article, MSE, RMSE, AIC and BIC, based on the properties of the distribution of the residuals.

It should be mentioned that for the ISS04 there is no established and statistically reliable way to perform the same evaluation and therefore the uncertainties of its estimates, particularly out of sample - that is extrapolation- are particularly problematic. This is another advantage of the proposed methodology, which should not be underestimated, because in general one of the main objectives of empirical scaling laws is to provide guidance to the planning of new experiments and the design of new devices.

3 The International Stellarator Confinement Database

For the studies reported in this paper, the largest database publicly available has been used [20]. It comprises entries from nine different machines: ATF, CHS, Heliotron E, Heliotron J, HSX, LHD, TJ-II, W7-A and W7-AS. These are all the devices deemed capable of providing reliable inputs at sufficiently high temperature over a reasonable range of parameters. Overall, the same variables and selection criteria, used in [5] to derive the ISS04 scaling, have been adopted in this work (ISHCDB 26). This is in harmony with previous studies and is motivated by specific experiments [21, 22] carried out in the past. For the reader's convenience, the main characteristics of the database and the notation are summarised in Table I.

A few cautionary remarks are in place, to make explicit the assumptions and approximations behind the analysis performed. First, it should be mentioned that the dependence of τ_E on the rotational transform is not easy to derive from the DB. Indeed the devices of the heliotron/torsatron family typically present a very strong collinearity between the rotational transform $t_{2/3}$ and the aspect ratio, due to engineering constraints. W7-A and W7-AS, on the other hand, do not scan a significant range of $t_{2/3}$. Therefore an approach coherent with the one proposed in [5] has also

Table I. The main entries in the database and their range: *n* is the average density and *B* the magnetic field on axis

Quantity	$([\min(°), \max(°)], [\mu, \sigma])$
$a[m]$	$([0.088, 0.634], [0.23, 0.12])m$
R[m]	$([0.938, 3.821], [1.94, 0.68])m$
P[MW]	$([0.04, 6.52], [1.09, 1.35])MW$
$n_e[10^{19}m^{-3}]$	$([0.22, 34.31], [5.42, 7.40])10^{19}m^{-3}$
$B[T]$	$([0.44, 2.56], [1.37, 0.63])10^{19}m^{-3}$
$t_{2/3}$	$([0.092, 1.607], [0.73, 0.44])$

been adopted to perform the analyses reported in this paper. The dependence of the energy confinement time on the rotational transform is determined by TJ-II, which is the device where this parameter has been scanned over the largest range. The value obtained using TJ-II data is used as first guess when running symbolic regression. The final scalings therefore typically show a power law dependence from $t_{2/3}$ with an exponent not much different from the one based on the TJ-II entries.

Also worth noting is the fact that the DB includes entries from discharges with different heating schemes, neutral beam and electron cyclotron resonance. The underlying hypothesis, already assumed in the derivation of ISS04, is therefore that the heating mechanism does not influence dramatically the confinement properties of the plasma. The entries available are not sufficient to derive robust scaling laws for each type of additional heating scheme. The same applies to plasma wall interactions: the database contains diverted plasmas and limited plasmas, and a variety of types of wall conditioning. On the other hand, all the devices in the DB have graphite plasma facing components.

Another important aspect to consider relates to the objective of the analysis. In addition to improving the understanding of the physics, the ambition is to provide a reliable scaling law in support to the planning of better experiments and the design of new devices. The main interest is therefore in the dependence of the energy confinement time on the main "engineering quantities",

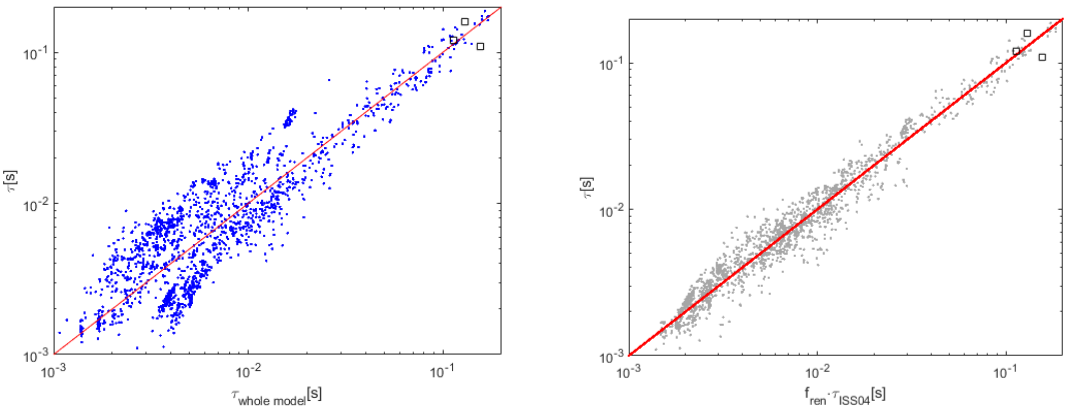

Figure 2. The log-log plots for the comparison of the models and the data. Left: the global scaling law without any rescaling factor (equation 3). Right: the power law ISS04 reported in Equation 2. The black squares are the predictions, assuming f_{ren} =1, for three different plasma configurations of W7-X (see Section 7).

namely minor and major radius, magnetic field and plasma density. In this respect, it is considered particularly relevant to analyse critically the estimates of traditional power laws, which are basically the only mathematical form of the models investigated so far.

4 The performance of unified scaling laws

The most recent and widely accepted scaling law for the energy confinement time in stellarators is the so-called ISS04 [5]. It has been obtained with log regression, adding to the set of regressors a dummy variable, which is to be considered a renormalization factor f_{ren}, specific to each device or even to different ranges of the operational parameters in the same machine. This renormalization coefficient has been introduced to take into account, among other features, the shear and therefore to improve previous scalings (ISS95). The ISS04 scaling law reads:

$$\tau_E^{ISS04} = 0.134\ a^{2.28} R^{0.64} P^{-0.61} n_e^{0.54} B^{0.84} t_{\frac{2}{3}}^{0.41} \qquad (2)$$

Table II Comparison of ISS04 with the non-power law scaling for the entire database. k indicates the number of parameters in the equations. MSE and RMSE are the Mean Squared Error and the Root Mean Squared Error of the residuals respectively.

Eq	k	MSE $[s^2]$	RMSE[s]	AIC	BIC
$\boldsymbol{\tau_E^{ISS04}}$	8	$0.15 \cdot 10^{-4}$	$0.39 \cdot 10^{-2}$	$-1.90 \cdot 10^4$	$-1.90 \cdot 10^4$
$\boldsymbol{\tau_E^{SR\ whole\ DB}}$ $\boldsymbol{Equ.(3)}$	17	$0.36 \cdot 10^{-4}$	$0.60 \cdot 10^{-2}$	$-1.76 \cdot 10^4$	$-1.76 \cdot 10^4$

The first question, arising naturally from the need to implement renormalization factors to obtain a global scaling law of good quality, is related to the rigidity of power law monomials. More flexible mathematical forms of the scaling could allow avoiding the need of specific multiplicative coefficients, improving the generality of the models and providing them with the flexibility that they currently lack. The first runs of SR via GP have therefore been devoted to trying to find a general scaling law for the entire database. The equation based on the Pareto Frontier contains two main addends. After refinement of the coefficients for each of the two main configurations (see later), the general form retained is:

$$\tau_E^{SR\ whole\ DB} = (3.80_{3.60}^{4.01}) \cdot 10^{-2} \cdot \left[a^{2.53_{2.48}^{2.58}} R^{0.97_{0.94}^{1.01}} P^{-0.60_{-0.62}^{-0.58}} n_e^{0.45_{0.43}^{0.48}} B^{0.67_{0.63}^{0.70}} t_{\frac{2}{3}}^{0.50_{0.49}^{0.51}} \left(1 + e^{-\left(\frac{\left(\frac{a}{R}\right) - 0.19_{0.18}^{0.20}}{0.017_{0.015}^{0.022}} \right)^2} \right) \right] +$$

$$+ (5.94_{5.66}^{6.22}) \cdot 10^{-2} \cdot \left[a^{2.24_{2.23}^{2.25}} R^{0.64_{0.63}^{0.65}} P^{-0.66_{-0.67}^{-0.65}} n_e^{0.57_{0.57}^{0.58}} B^{1.03_{0.98}^{1.08}} t_{\frac{2}{3}}^{0.37_{0.35}^{0.38}} \frac{1}{\left(1 + 3.78_{3.43}^{4.14} e^{-\frac{2R}{R_{Av}}} \right)} \right] \quad (3)$$

Where R_{Av} has unit of length and a numerical value of 2.44, an average of the major radii in the DB.

The quality of model (3) is compared to the one of ISS04 in Table II, whose notation is the following. MSE and RMSE indicate the Mean Squared Error and the Root Mean Squared Error of the residuals respectively; as usual, the residuals are defined as the differences between the data and the predictions of the models. AIC and BIC are the model selection criteria mentioned in Section 2. All the indicators reported in Table II can be considered cost functions in the sense that the better a model the lower their value.

Various considerations can be derived from inspection of Equation (3) and Table II. First, the model derived by the deployment of SR via GP, even without any renormalization factor, is quite competitive with ISS04, according to all the statistical indicators; indeed, the performances of model (3) are just slightly worse than those of ISS04. In graphical form, the quality of the new non-power law scalings can be appreciated from the traditional type of log-log plots reported in Figure 2. Inspection of Figure 2 reveals first the inadequacies of this type of plots, which can be very misleading. Indeed the statistical indicators reported in Table II show that the difference between equations (2) and (3) are much less pronounced than the visual appearance of the log-log plots would suggest. Moreover, the difficulties of obtaining a good quality of the fit are mainly due to the data of the shearless devices (see later and Appendix A). In terms of interpretation, equations (3) is not a power law; indeed, it contains exponential terms and is not a monomial.

Even if its performances are more than satisfactory, the mathematical form of equation (3) is quite complex and contains two different major terms. In this respect, it should be remembered that the database includes machines implementing two main types of configuration with respect to the rotational transform profile, with and without shear. The first class includes the devices ATF, CHS, HELE, HELJ, LHD. The shearless devices are W7A, W7AS and TJ-II. This distinction, already introduced in [22], is confirmed by visual inspection of the database, which reveals that, indeed, the dependencies of the energy confinement time on the regressors can be different for these two types of configurations. It has therefore been decided to particularise the scaling laws for these two different groups of machines. The scaling law obtained, using SR via GP for the magnetic configuration with shear, is:

$$\tau_E^{SR\ Shear} = (7.92_{7.73}^{8.11}) \cdot 10^{-2} \cdot$$

$$a^{2.53_{2.48}^{2.58}} R^{0.97_{0.94}^{1.01}} P^{-0.60_{-0.62}^{-0.58}} n_e^{0.45_{0.43}^{0.48}} B^{0.67_{0.63}^{0.70}} t_{\frac{2}{3}}^{0.50_{0.49}^{0.51}} \left(1 + e^{-\left(\frac{\left(\frac{a}{R}\right) - 0.19_{0.18}^{0.20}}{0.017_{0.015}^{0.022}}\right)^2}\right) \qquad (4)$$

A comparison of the statistical performance of this non-power law scaling with the ISS04 is reported in Table III. For this model, the traditional log-log plot of the scaling quality is shown in Appendix A, where it is also compared with the ISS04 for this subset of the database. As can be

Table III Comparison of ISS04 and the non-power law scalings for the machines with and without shear.

Eq	k	MSE $[s^2]$	RMSE[s]	AIC	BIC
$\tau_E^{ISS04\ Shear}$	8	$2.64 \cdot 10^{-5}$	$5.14 \cdot 10^{-3}$	$-8.00 \cdot 10^3$	$-7.96 \cdot 10^3$
$\tau_E^{SR\ shear}$ $\boldsymbol{Equ.}(\mathbf{4})$	9	$2.16 \cdot 10^{-5}$	$4.70 \cdot 10^{-3}$	$-8.16 \cdot 10^3$	$-8.11 \cdot 10^3$
$\tau_E^{ISS04\ Shearless}$	8	$6.43 \cdot 10^{-6}$	$2.50 \cdot 10^{-3}$	$-1.14 \cdot 10^4$	$-1.14 \cdot 10^4$
$\tau_E^{SR\ shearless}$ $\boldsymbol{Equ.}(\mathbf{5})$	8	$18.69 \cdot 10^{-6}$	$4.32 \cdot 10^{-3}$	$-1.04 \cdot 10^4$	$-1.05 \cdot 10^4$

derived from simple inspection of Table III, equation (4) presents slightly better performance than ISS04 according to all statistical indicators, even without the use of any renormalization factor.

A cautionary remark is in place, given the quite different optimization level of the machines with shear, particularly LHD. It is indeed well known that LHD at R=3.60m is neoclassically optimized w.r.t. to LHD at R=3.90 (and other devices are optimised at different degrees). Since neoclassical transport is important in stellarators, the effectiveness of a unified scaling law could be considered surprising. The underlying hypothesis in the present analysis is that the differences in neoclassical transport, between the configurations in the databse, are less relevant than the contribution of turbulence to the energy confinement. Furthermore, neoclassical transport is often

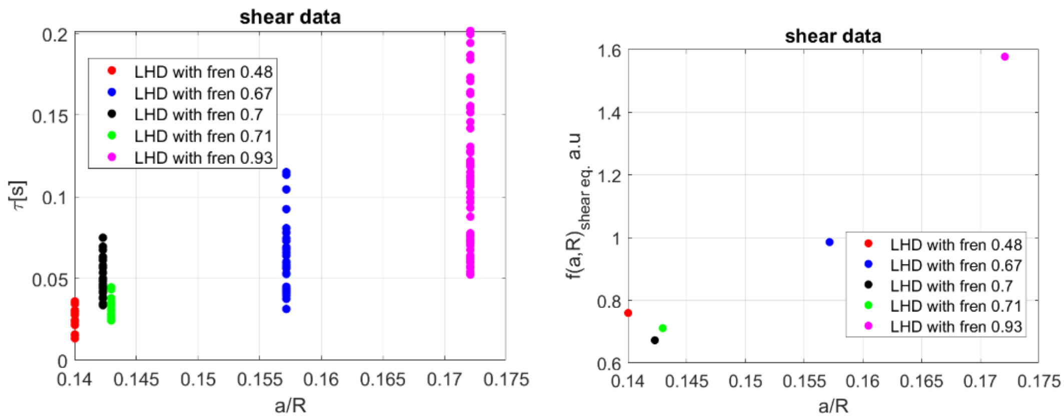

Figure 3. Left: variation of τ_E vs a/R for different optimization levels in LHD. Right: trend of the term *f(a,R)*, function of minor radius *a* and major radius *R*, in the scaling law (4). The effects of optimization on the other 0D quantities are reported in Appendix B.

more important in the “hot core”, whereas the energy confinement time is strongly weighting the edge, because its volume is larger. In addition, none of the existing stellarators has been optimized with respect to turbulent energy transport, and therefore it is reasonable to expect that their turbulent transport is not determined by its 3D characteristics, but by the 0D parameters used in this work. In any case, it should be noticed that the optimization of LHD has implied significant variations also of the 0D quantities, as can be seen in the plots of Figure 3 and Appendix B. From these plots, it is easy to see how equation (4) can fit the trends of the optimisation level equally well or even better than the normalization factor f_{ren} in the ISS04. It is therefore no surprising that the SR via GP tools, with their high exploratory capability, manage to identify models, which can reproduce quite well the entries at different optimization levels on the basis of only 0D quantities. Of course, to really capture all the higher dimensional effects, the proposed methodology will have to be applied to more sophisticated DBs, including at least the main plasma profiles (efforts in this direction are already underway).

The scaling law obtained for the shearless magnetic configuration using SR via GP is:

$$\tau_E^{SR\ Shearless} = (11.23_{10.69}^{11.78}) \cdot 10^{-2} \cdot$$
$$\cdot\, a^{2.24_{2.23}^{2.25}} R^{0.64_{0.63}^{0.65}} P^{-0.66_{-0.67}^{-0.65}} n_e^{0.57_{0.57}^{0.58}} B^{1.03_{0.98}^{1.08}} t_{\frac{2}{3}}^{0.37_{0.35}^{0.38}} \frac{1}{\left(1 + 3.78_{3.43}^{4.14} e^{-\frac{2R}{R_{Av}}}\right)} \qquad (5)$$

A comparison of the statistical performance of this non-power law scaling with the ISS04 is reported in Table III as well. Also for this model, the traditional plots of the scaling quality are reported in Appendix A. In this case, equation (5) presents slightly worse performance than ISS04 according to all statistical indicators. This is due to the heterogeneous character of this part of the database, which in reality contains data from two different configurations; indeed TJ-II is a heliac, while W7-A is a classical stellarator, of which W7-AS can be considered an optimized version. Unfortunately, the database does not provide enough entries to derive robust different models for the two types of magnetic configuration.

Equations (4) and (5) are not in power law form, because they contain exponential and squashing terms, which are essential to obtain good performance. It is probably worth stating again that the non-power law scalings have been obtained without making recourse to any rescaling factor. The obtained quality of the models, shown in Table III, is therefore quite remarkable, since the renormalization factor, used to obtain ISS04, varies over a very wide interval, from 0.25 to 1.

The exponential and squashing terms in the scaling laws require some comments. They are indispensable to obtain a good fit of the database, because the smaller devices present different trends compared to the larger ones. The power laws are too rigid to accommodate this difference properly and a reasonable compromise cannot be found. This characteristic of the database is the main reason why a renormalization factor had to be implemented to obtain the ISS04. Getting rid of the power law constraint, with the help of SR via GP, is an alternative to introducing a renormalisation coefficient for increasing the flexibility of the scalings. The plots supporting this interpretation are reported in Appendix C, in which the effects of not considering the exponential terms are particularised for the individual devices. It should also be mentioned that the same problems of global consistency affects also the Tokamak configuration, as was emphasised for the case of the ITPA database in [7-9].

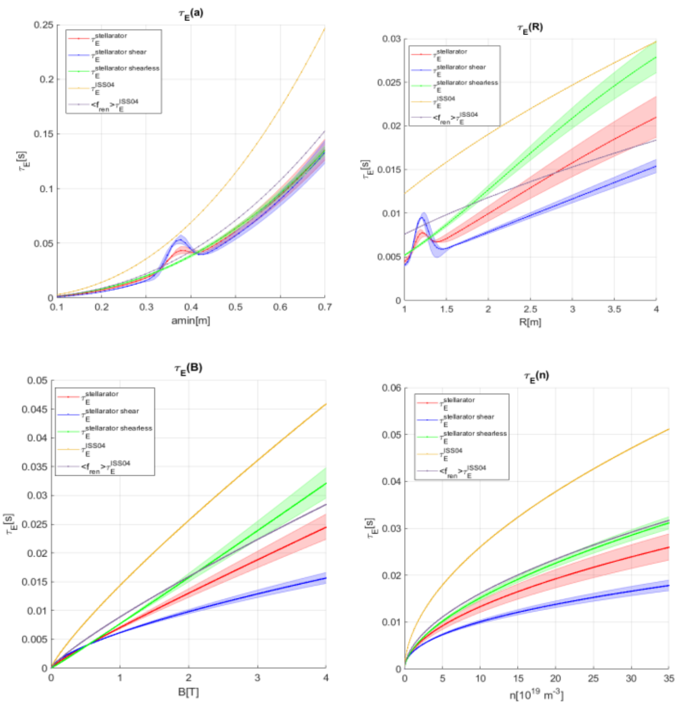

Figure 4. The trends of the equations obtained with SR via GP compared to the ISS04. For the ISS04, two scalings are shown: one with a renormalization factor equal to 1 and one (indicated with <…>) with the renormalization factor averaged over the entries in the DB.

Another positive aspect of not constraining the scalings to be power law monomials is that the obtained flexibility allow fitting quite well also the data of the individual devices. The quality of the agreement can be appreciated again by inspection of the plots of Appendix C.

The trends of the ISS04 and the three ones obtained with SR via GP are compared in Figure 4 over the interval covered by the machines in the database. The scaling of the machines with shear seems to have a different dependence on the main engineering quantities than the shearless one. On the other hand, again caution is appropriate in interpreting these visual representations of the scalings. Indeed, for clarity sake, in these plots only one parameter is scanned at the time, keeping all the others fixed at the value corresponding to their average in the database.

5 Summary, discussion and lines of further investigations

In this paper, it has been investigated how symbolic regression via genetic programming can constitute an alternative approach to the extraction of scaling laws for the energy confinement time from the international Stellarator database. The rationale behind this strategy is to verify whether relaxing the power al constraint, with the help of SR via GP, can provide enough flexibility to the mathematical form of the scalings, so that the use of renormalisation factors is avoided. With this approach, a general non power law equation for the entire database, without any renormalisation factor and competitive with ISS04, has been identified. On the other hand, the exploratory use of SR via GP has revealed that significantly better performance can be obtained by particularising the scalings for the shear and shearless configurations. Indeed, a priori, there is no major reason why the two configurations should scale exactly in the same way towards the parameter region of the reactor.

As already shown also for the Tokamak configuration, power laws are not necessarily the most appropriate form for the scalings [6-13]. Atomic physics aspects and symmetry breaking limit the validity of the self-similarity assumption; therefore, power laws become unsatisfactory. Relaxing the power law constraint introduces enough flexibility in the models to obtain the same of better statistical properties without any renormalization factors. Moreover, it is also important to notice that the scaling laws obtained with SR via GP reproduce quite well also the trends of the energy confinement time in the individual devices (see Appendix C). This is not the case of many traditional scalings, including those for the Tokamak, which provide only good cross-machine predictions. This is another piece of evidence supporting the opinion that power laws can lack flexibility to investigate a complex physical quantity such as τ_E.

Table IV. First estimates of τ_E for W7-X. In the first column the values in brackets indicate the intervals of the energy experimental confinement time reported in [23].

Fuchert et al [25] [ms]	Shearless [ms]	ISS04 [ms]
A[70~110]	180_{166}^{195}	179_{167}^{192}
B[100~120]	133_{121}^{146}	136_{126}^{145}
D[150~160]	155_{140}^{170}	157_{145}^{170}

The absence of renormalisation factors in the scalings is important also for interpretability and physical understanding. Indeed, SR via GP can pick up the effects of the Stellarator optimization that leave a signature in the 0D quantities contained in the database, which it can do very well as shown in Figure 3 and Appendix B. Of course, being fully data driven, the tools deployed to obtain the results reported in this paper are absolutely agnostic about the information not available in the DB. On the other hand, it should be considered that the scaling laws, derived with SR via GP, basically confirm the expectations of the

ISS04, employing f_{ren} = 1, for the largest present day devices. Since the mathematical methods used are completely independent, the agreement in the final estimates, using a realistic value of f_{ren}, increases significantly the confidence in the results. If one considers that in [5] the difference in f_{ren} between configurations was a factor of 5, contributing to narrowing down the value of f_{ren} for new experiments can be of obvious relevance.

In any case, specific experiments in present machines will have to be designed to confirm the main trends derived from the available database, which is quite limited particularly for extrapolation to future devices. In this perspective, extremely important would the new data of W7-X, which was not represented in the version of the database used to obtain the ISS04. In this perspective, the predictions of the scaling law for the shearless devices, equation (5), seem to be encouraging; they are in quite good agreement with the estimates of ISS04, as reported in Table IV, and with the results reported in [23]. The engineering parameters, used to obtain the values reported in Table IV, are a=0.49m, R=5.5m, $t_{2/3}$=0.8, B=2.5T. The three experimental configurations A, B and D (see [27]) correspond to the following values of the other required quantities, indicated with the notation Config: $(n[10^{19}m^{-3}], P[MW])$, $A = (0.6, 0.5), B = (1.25, 1.5)$, $D = (2.25, 2)$. These three experimental points are reported as squares in the log-log plots of Figure 2, to visualise the level of extrapolation required to predict them. Again, the values from ISS04 have been derived by setting the renormalization factor equal to 1. Both scalings, ISS04 and the one obtained with SR via GP, tend to slightly overestimate the experimental values but this is understandable, since the W7-X plasmas are from the initial limiter campaign, where radiation losses were significant (ISS04 is strictly valid for low-radiation plasmas). Especially case A has a clearly degraded confinement [23].

In terms of physical interpretation, it should be remembered that the motivation, behind the attempt to identify a single scaling law without f_{ren}, is also to test whether all Stellarators follow experimentally a similar trend, because differences in neoclassical transport levels are not so important and, together with the differences in turbulent transport, can be basically captured also by a more sophisticated analysis of the 0D quantities. The positive results obtained with SR via GP could hint towards turbulence being a relevant transport mechanism. It may be that turbulence and neoclassical transport interact in a way, which is not so configuration-dependent, when considering the two big families of magnetic fields topologies, with and without shear. For instance, the total energy transport of Wendelstein 7-X has been observed to be less configuration dependent than the neoclassical contribution.

With regard to the mathematical form of the scalings, it should be mentioned that theoretical considerations may lead to monomial power laws in pure regimes, in which only one mechanism, neoclassical or turbulent, is responsible for the global transport [23]. On the other hand, as soon as a combination of these regimes is considered [24], power laws become too rigid [25]. Also in cases where neoclassic transport dominates, power laws could be insufficiently flexible because self-similarity can be broken by non-plasma physics effects, such as those due to atomic physics at the edge. Evidence of the insufficiencies of power laws for the Tokamak is increasing quite rapidly and can be justified also in terms of broken symmetries and differences in the current profiles [6-9].

Notwithstanding the positive aspects just mentioned, excessive confidence in the predictive capability of the obtained scaling laws should be moderated by a few considerations. First, of course, the entire exercise hinges on the assumption that the physics remains unchanged in the "out of sample" operational space. Moreover, the database available remains quite unsatisfactory. The range of parameters scanned is not very large and the number of entries quite low for a complex system such as a Stellarator, whose energy confinement time depends at least on the six quantities included in the ISS04. Moreover, in order to fully take into consideration the details of the optimisation all the relevant quantities, including profiles, will have to be included in the DB, irrespective of the numerical tools adopted to derive the scalings [26].

In addition to the cautionary notes about the statistical properties of the DB, some aspects related to the physics have also to be mentioned. First, the devices, whose inputs are included in the DB, have all graphite plasma facing components. The experience of AUG and JET with metallic walls indicates that the wall materials can affect various aspects of the confinement in not negligible way; a reassessment, when Stellarators are operated with a metal wall, will have to be undertaken. The entries of the DB also comprise discharges with quite low radiated fraction. The demonstrative reactors are expected to operate at very high radiation, well above 90 % radiated fraction, and it is not obvious that this can be achieved without any detrimental effect on confinement [27-30]. Changes in the isotopic composition do not seem to have great impact on the Stellarator properties but this remains to be fully confirmed, since this result seems to contradict the evidence for the Tokamak configuration.

Aknowledgements

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 and 2019-2020 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission. This work was also partially funded by the Spanish Ministry of Economy and Competitiveness under the Project No. ENE2015-64914-C3-1-R

References

[1] J.Wesson "*Tokamaks*" Oxford University Press 2011

[2] Wakatani, M. (1998). Stellarator and Heliotron Devices. *Oxford University Press.* ISBN 978-0-19-507831-2.
[3] McDonald D. *et al* 2004 *Plasma Phys. Control. Fusion* **46** 519–34
[4] http://efdasql.ipp.mpg.de/hmodepublic/DataDocumentation/ Datainfo /DB3v13/db3v13.html
[5] Yamada H. et al Nucl. Fusion **45** (2005) 1684–1693
[6] Murari, A. et al. "*Non-power law scaling for access to the H-mode in tokamaks via symbolic regression*" (2013) Nuclear Fusion, 53 (4), art. no. 043001, DOI: 10.1088/0029-5515/53/4/043001
[7] Murari A. et al., P. "*Symbolic regression via genetic programming for data driven derivation of confinement scaling laws without any assumption on their mathematical form*" (2015) Plasma Physics and Controlled Fusion, 57 (1), art. no. 014008, DOI: 10.1088/0741-3335/57/1/014008
[8] Murari A. et al "*A new approach to the formulation and validation of scaling expressions for plasma confinement in tokamaks*" 2015 Nuclear Fusion, Volume 55, Number 7 https://doi.org/10.1088/0029-5515/55/7/073009
[9] Murari A. et al, "*Application of symbolic regression to the derivation of scaling laws for tokamak energy confinement time in terms of dimensionless quantities*" Nuclear Fusion **56**, 2, 26005, DOI: 10.1088/0029-5515/56/2/026005
[10] Murari A. *et al* "*Clustering based on the geodesic distance on Gaussian manifolds for the automatic classification of disruptions*" 2013 Nucl. Fusion **53** 033006, doi.org/10.1088/0029-5515/53/3/033006
[11] Peluso E.et al. "*A Statistical Analysis of the Scaling Laws for the Confinement Time Distinguishing between Core and Edge*", Physics Procedia, Volume 62, 113-117, (2015), doi: dx.doi.org/10.1016/j.phpro.2015.02.020
[12] Peluso E. et al, *"A statistical method for model extraction and model selection applied to the temperature scaling of the L-H transition"* (2014) Plasma Physics and Controlled Fusion, 56 (11), art. no. 114001
[13] Murari A. et al, *"Robust scaling laws for energy confinement time, including radiated fraction, in Tokamaks",* Nucl.Fusion Vol.57, num.12 (2017)
[14] Schmid M and Lipson H, Science, Vol 324, April 2009
[15] Koza J.R., "*Genetic Programming: On the Programming of Computers by Means of Natural Selection"*. MIT Press, Cambridge, MA, USA (1992).
[16] Burnham K.P. and Anderson D.R. 2002 "*Model Selection and Multi-Model Inference: a Practical Information-Theoretic Approach"* 2nd edition (New York: Springer)
[17] Kaisa Miettinen (1999). *Nonlinear Multiobjective Optimization*. Springer. ISBN 978-0-7923-8278-2.
[18] Bates D and Watts D 1988 *Nonlinear Regression Analysis and Its Applications* (New York: Wiley)
[19] Coleman T F and Li Y 1996 SIAM J. Optim. 6 418–45, Coleman T F and Li Y 1994 Math. Program. 67 189–224
[20] https://ishpdb.ipp-hgw.mpg.de/
[21] https://ishpdb.ipp-hgw.mpg.de/ISS/documentationDB07_26.pdf

[22] U.Stroth et al "*Energy confinement scaling from the international stellarator database*" Nuclear Fusion, Volume 36, Number 8 1996

[23] G.Fuchert et al "*Global energy confinement in the initial limiter configuration of Wendelstein 7-X*" 2018 Nucl. Fusion 58 106029 https://doi.org/10.1088/1741-4326/aad78b

[24] A.Dinklage et al Nuclear Fusion, Volume 47, Number 9 (2007) "*Physical model assessment of the energy confinement time scaling in stellarators*" https://iopscience.iop.org/article/10.1088/0029-5515/47/9/025

[25] N.Pablant et al Nuclear Fusion, Volume 60, Number 3 (2020) "*Investigation of the neoclassical ambipolar electric field in ion-root plasmas on W7-X*" https://iopscience.iop.org/article/10.1088/1741-4326/ab6ea8

[26] Dinklage, A.; et al "*Assessment of Global Stellarator Confinement: Status of the International Stellarator Confinement Database*" Fusion Science and Technology, Taylor & Francis, 2007, 51, 1-7 https://doi.org/10.13182/FST07-A1281

[27] Otto J W F Kardaun "*Interval estimation of global H-mode energy confinement in ITER*" 1999 *Plasma Phys. Control. Fusion* 41 429

[28] Murari A. and Peluso E. et al., *"Investigating the thermal stability of highly radiative discharges on JET with a new tomographic method",* Nucl. Fusion 60 046030, doi: https://doi.org/10.1088/1741-4326/ab7536

[29] G. Fuchert et al., Nucl. Fusion 60, (2020) 036020 "*Increasing the density in Wendelstein 7-X: benefits and limitations*" https://doi.org/10.1088/1741-4326/ab6d40

[30] R. Preuss, A. Dinklage, and A. Weller "*Energy-Confinement Scaling for High-β Plasmas in the W7-AS Stellarator*" Phys. Rev. Lett. 99, 245001 December 2007

Appendix A. Comparison of the power law and non-power law scalings.

This appendix reports the log/log plots of the experimental energy confinement time versus the predictions of the various scaling laws. The plots are particularised for the configurations with and without shear. As usual, caution must be exerted in drawing conclusions from the visual appearance of the plots, which can be misleading. Reference to the statistical indicators reported in Section 4 is always recommended.

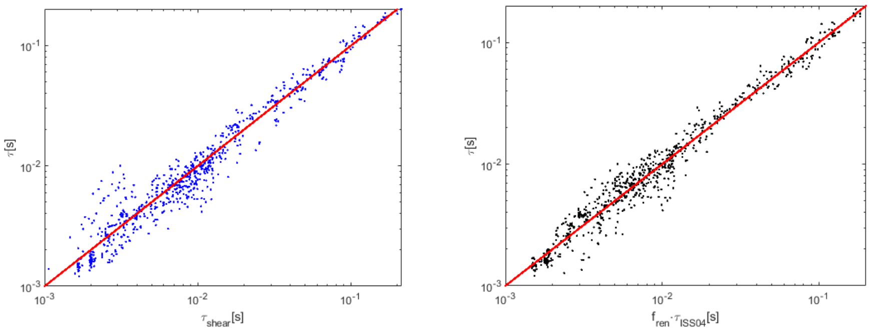

Figure A1. The log-log plots for the comparison of the models and the data for the devices with shear. Left: the scaling law obtained with SR via GP without any rescaling factor (equation 4). Right: the power law ISS04 reported in Equation 2.

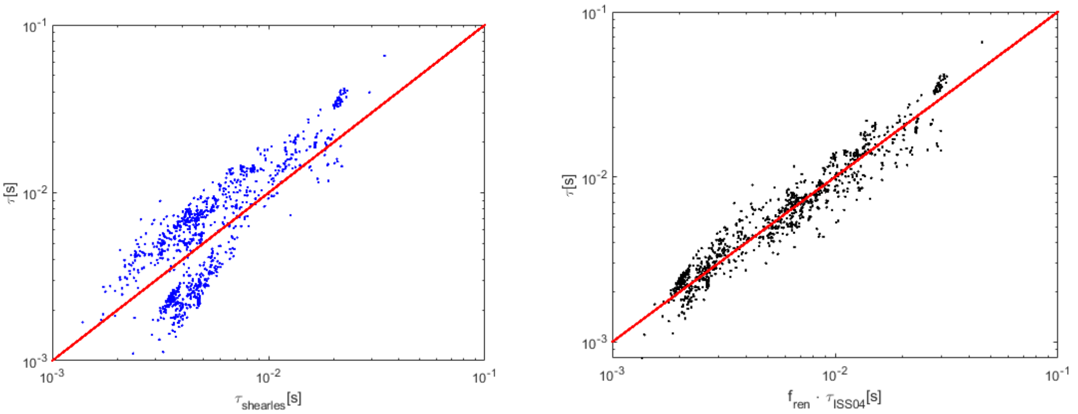

Figure A2. The log-log plots for the comparison of the models and the data for the shearless devices. Left: the scaling law obtained with SR via GP without any rescaling factor (equation 5). Right: the power law ISS04 reported in Equation 2.

Appendix B. LHD specific analyses.

This appendix is devoted to assessing the quality of the derived scaling for the devices with shear, using LHD data at different optimisation levels (different f_{ren}) and data not included in the set adopted for the derivation of ISS04. First, the quality of the fits at the various levels of optimization are reported (Figures B1 to B3). Then, in Figure B4, it is shown how the scaling obtained with SR via GP predicts quite well the points at high beta not included in the data used to obtain the scalings.

In figures B1-B3, the predictions for LHD are plotted vs minor radius, major radius and iota. The

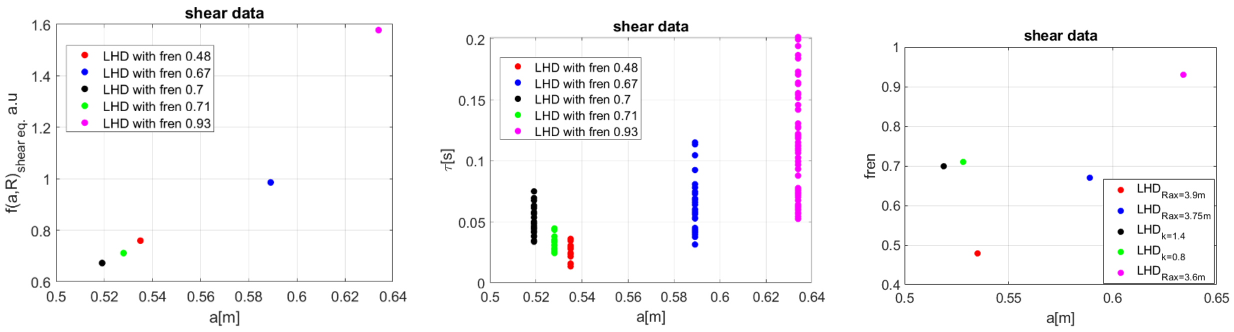

Figure B1 Analysis of LHD data for various optimisation levels vs the minor radius. Left: The part of the model equation (4), which depends on major and minor radius *f(a,R)*. Centre: the actual confinement time in LHD vs the minor radius. Right: trend of f_{ren} in ISS04 vs the minor radius.

trends of τ_E are compared with the part of the model equation (4), which depends on major and minor

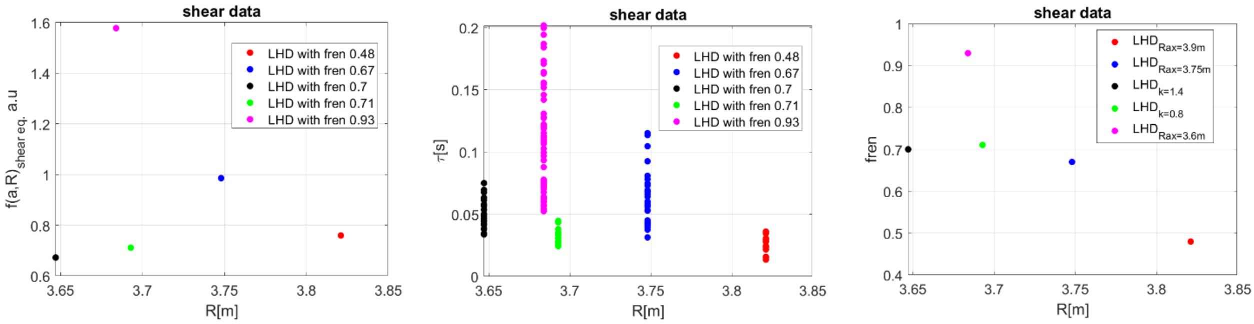

Figure B2 Analysis of LHD data for various optimisation levels vs the major radius. Left: The part of the model equation (4), which depends on major and minor radius *f(a,R)*. Centre: the actual confinement time in LHD vs the major radius. Right: trend of f_{ren} in ISS04 vs the major radius.

radius *f(a,R)*. The trends of the renormalization factor f_{ren} with the geometric quantities and iota are also reported. From the plots, it is easy to see how the scaling of equation (4), obtained with SR via GP, can reproduce the effects of the various optimization levels on the 0D quantities equally well, if not better, than the renormalization factor f_{ren}.

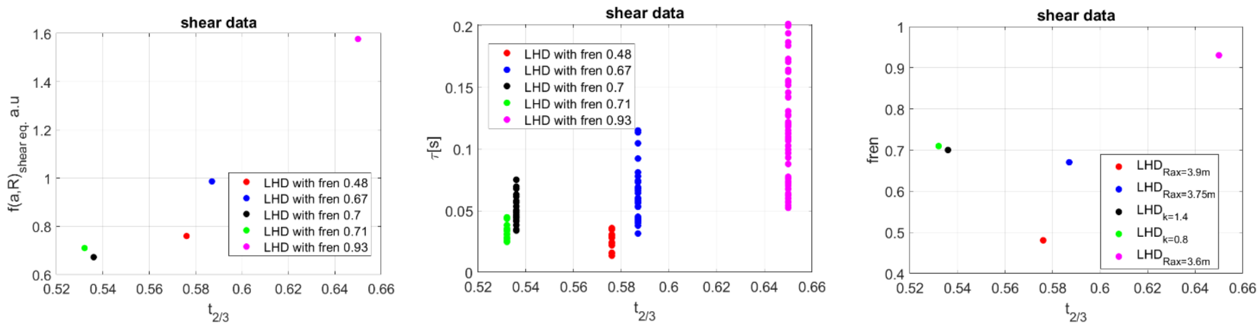

Figure B3 Analysis of LHD data for various optimisation levels vs iota. Left: The part of the model equation (4), which depends on major and minor radius *f(a,R)*. Centre: the actual confinement time in LHD vs iota. Right: trend of f_{ren} in ISS04 vs iota.

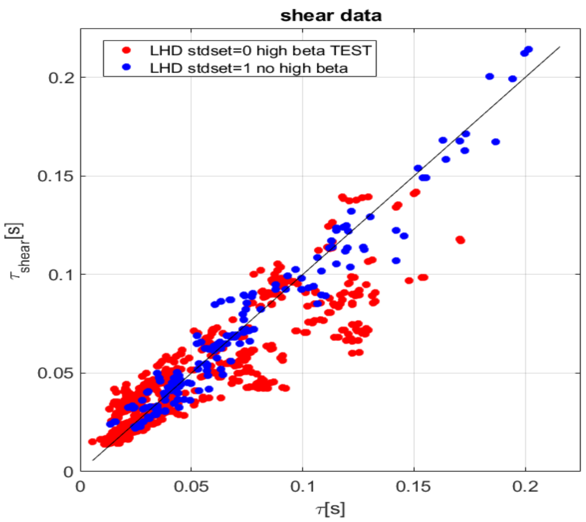

Figure B4 LHD data. In blue entries included in the analysis in the main paper and selected using the STDSET=1 flag. In red, those used as "test", selected using the STDSET=0 flag, and therefore related to operation at high beta. The shear model equation (4), derived including all the devices with shear, fits well also the high beta LHD data.

Finally, as can be seen from Figure B4, the scaling for the shearless devices (equation 4) seems to fit quite well also the entries of the database at high beta [32]. These are points not included in the set used to derive either equation (4) or ISS04 and are therefore to be considered a test set, in the language of machine learning.

Appendix C. Effect of the non-power law terms in the scalings.

This appendix reports the log/log plots of the predictions of the non-power law scalings versus the experimental energy confinement time. The empty symbols indicate the predictions of the power law part of the non-power law scalings, showing the importance of the exponential terms to fit the experimental data. Including the non power law terms always improves the fit of the entries in the database.

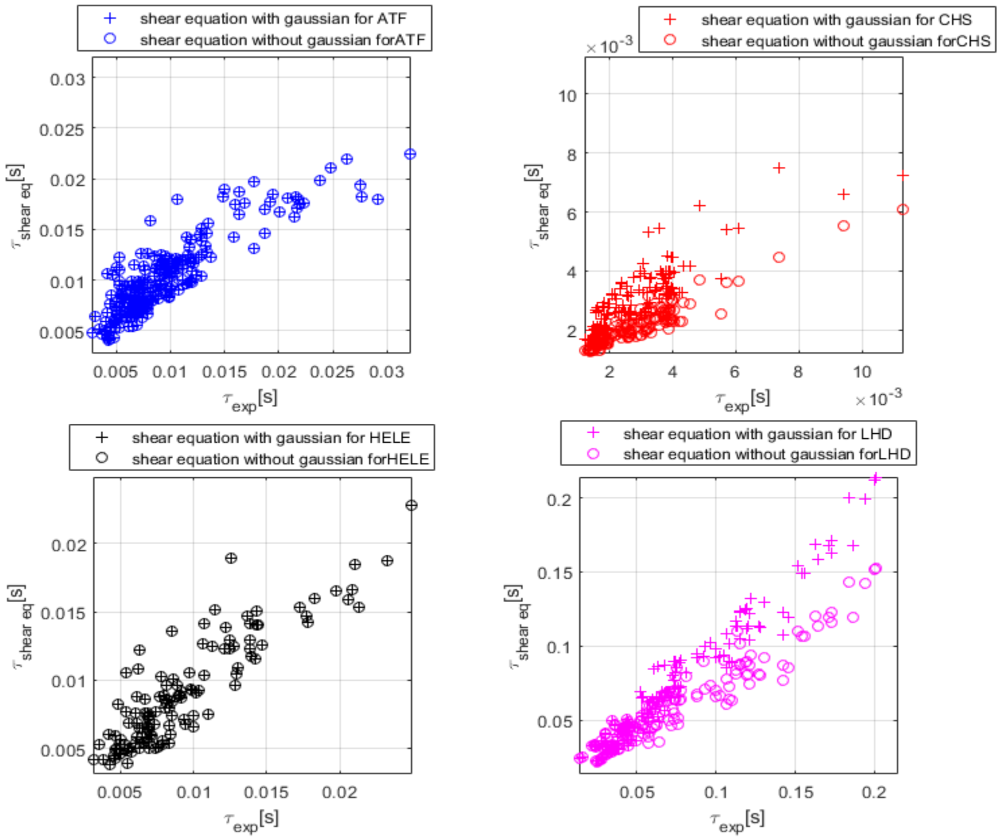

Figure C1. Comparison of the scaling laws with and without the exponential term for the devices with shear.

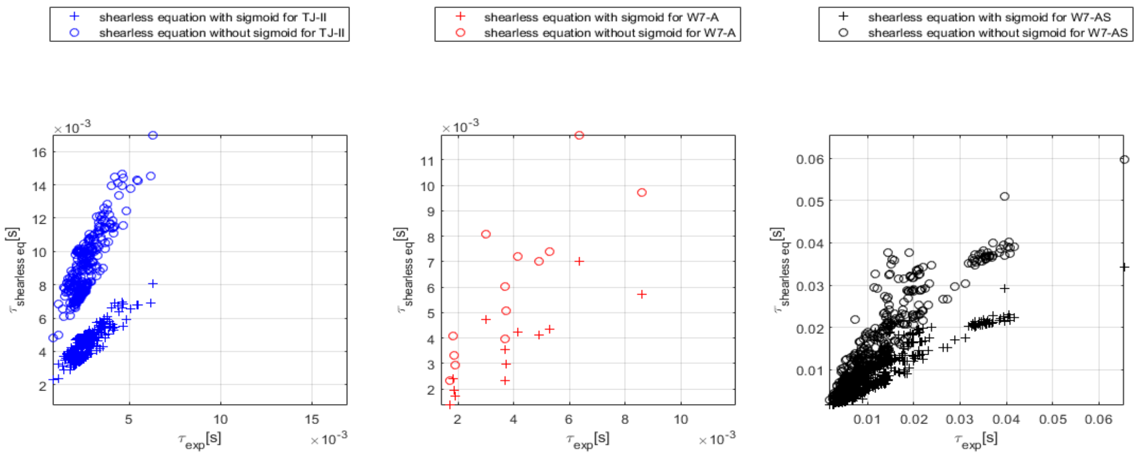

Figure C2. Comparison of the scaling laws with and without the exponential term for the shearless devices.