Coulomb Energy Differences in $T = 1$ Mirror Rotational Bands in ^{50}Fe and ^{50}Cr

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Gamma rays from the $N = Z - 2$ nucleus ^{50}Fe have been observed, establishing the rotational ground state band up to the state $J^{\pi} = 11^{+}$ at 6.994 MeV excitation energy. The experimental Coulomb energy differences, obtained by comparison with the isobaric analog states in its mirror ^{50}Cr, confirm the qualitative interpretation of the backbending patterns in terms of successive alignments of proton and neutron pairs. A quantitative agreement with experiment has been achieved by exact shell model calculations, incorporating the differences in radii along the yrast bands, and properly renormalizing the Coulomb matrix elements in the pf model space.

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The charge independence of the strong interaction is a fundamental assumption in nuclear physics. This symmetry between protons and neutrons means that they can be viewed as two states of the same particle, the nucleon, characterized by different projections of the isospin quantum number. Experimentally, the isospin symmetry shows as nearly identical spectra in pairs of mirror nuclei (obtained by interchanging protons and neutrons). The Coulomb field splits the isobaric multiplets very much as the magnetic field splits the angular momentum multiplets while respecting the underlying symmetry. In the case of isospin, the expected breakdown is small because the large Coulomb effects depend only on Z—the number of protons—which is a conserved quantity. As a result, the contributions to the bulk energy, which go as Z^2 and can be of hundreds of MeV, conserve isospin. The same applies to the displacement energies between analog ground states, which vary as Z and can be as large as 10–20 MeV. When we come to Coulomb energy differences (CED) between *excited* analog states in mirror nuclei, the effects become so small (10–100 keV) that they can be treated in first order perturbation theory, i.e., by taking the expectation value of the Coulomb potential between nuclear wave functions of *good isospin*. Given that large effects have been eliminated in the CED, and that the wave functions remain essentially unperturbed, the Coulomb field becomes a delicate probe of *nuclear* structure.

To understand what is being probed there exists one privileged region: The $1f_{7/2}$ shell, where backbending rotors have been observed [1,2], and where—quite conveniently—the necessary experimental and theoretical tools have become available and improved simultaneously in the last few years.

Previous studies of CED in odd-mass ($T = 1/2$) rotating mirror nuclei in the $1f_{7/2}$ shell indicate that the alignment mechanisms invoked to explain backbending [3,4] lead to a qualitative understanding of the observed energy patterns [5–7]. Quantitatively, the severe test of our understanding of Coulomb effects provided by CED has not yet been fully passed, though significant advances have been made.

In this work we report the first investigation of high spin structure in the $N = Z - 2$ nucleus ^{50}Fe. Together with ^{50}Cr it constitutes the heaviest $T = 1$ mirror pair studied so far and the first case in which $T = 1$ rotational bands can be investigated at the backbending region. ^{50}Cr is known to be one of the best rotors in the $f_{7/2}$ shell with a deformation parameter $\beta \approx 0.25$ [8–10]. So far only the ground state and an excited $J = 6$, $T = 2$ state were known in ^{50}Fe from beta decay studies. To study the high spin states we have performed an experiment with the EUROBALL array. The nucleus ^{50}Fe was produced in the reaction ^{28}Si + ^{28}Si at 110 MeV bombarding energy, after the evaporation of one α particle and two neutrons. The beam was delivered by the XTU Tandem accelerator of the Legnaro National Laboratory. The target consisted of 0.8 mg/cm^2 of ^{28}Si (enriched to 99.9%) with a 15 mg/cm^2 gold backing. Gamma rays were detected with the EUROBALL array equipped with 26 clover and 15 cluster composite (Compton suppressed) detectors, which provided a photo-peak efficiency of about 7%. The forward 1π solid angle was covered by the neutron wall (NW) [11], consisting of 50 liquid scintillator neutron detectors. Charged particles were detected with the ISIS array [12] consisting of 40 E-ΔE telescopes. Events were collected when *either* at least three Ge detectors plus one detector of the NW fired in coincidence *or* at least two Ge detectors fired and one neutron was pulse shape discriminated in coincidence in the NW.

Data were sorted into γ-γ matrices in coincidence with charged particles and neutrons. The efficiencies for charged particles were estimated, from the present data, to be about 50% for protons and 20% for alphas, while for neutrons, the resulting efficiency was about 20%. For the identification of candidate lines as belonging to ^{50}Fe, the requested condition was to be in coincidence with neutrons, only one α particle and not in coincidence with protons. A cascade of six γ lines was found to fulfill those requirements and was thus assigned to ^{50}Fe. These transitions were also observed in a γ spectrum in coincidence with two neutrons but in this case the statistics was not enough to construct a γ-γ matrix. After the efficiency correction, performed with ^{152}Eu and ^{56}Co sources, the cascade of six new γ rays was ordered in accordance with their relative intensities and assigned to the ground state rotational band of ^{50}Fe, analog to that in its mirror nucleus ^{50}Cr.

In Fig. 1a a spectrum corresponding to a sum of gates on the lowest three transitions assigned to ^{50}Fe is shown. It was obtained from a matrix constructed by adding a γ-γ matrix in coincidence with neutrons and one α particle plus another matrix coincident only with neutrons

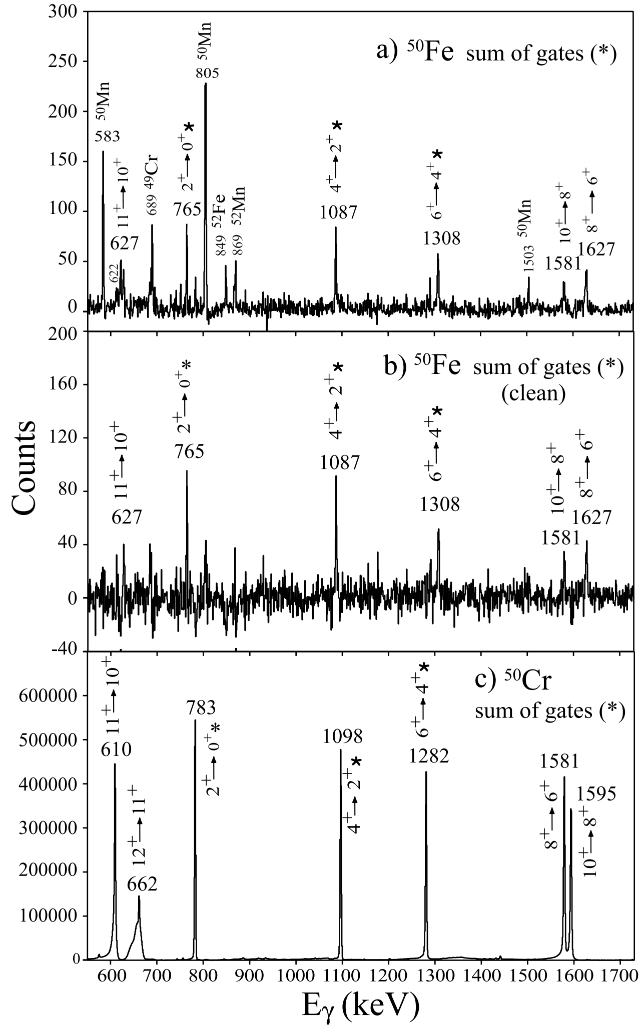

FIG. 1. γ-ray spectra for the $T = 1$ mirror pairs: sum of gates on the three lowest transitions for (a) ^{50}Fe; (b) same as (a) after the subtraction of the contaminating channels; and (c) ^{50}Cr.

(with a veto for any charged particle). The latter matrix takes into account those α particles not detected due to the low efficiency detection. Contaminating lines from strong channels, whose evaporated protons were not detected, are marked in the figure. A clean spectrum obtained after properly subtracting the contaminating channels is shown in Fig. 1b.

For comparison, the spectrum for ^{50}Cr ($\alpha 2p$ reaction channel) is shown in Fig. 1c. It was obtained as a sum of spectra gated on the first three analog transitions in ^{50}Cr in a γ-γ matrix in coincidence with one proton. Because of the different trigger conditions in the two ones, it is difficult to give a precise relative cross section. An estimated ratio of the cross sections is $\sigma(^{50}\mathrm{Cr})/\sigma(^{50}\mathrm{Fe}) \approx 10^4$.

The level schemes of the two $T = 1$ mirror nuclei are shown in Fig. 2. ^{50}Cr has been extensively studied both theoretically and experimentally [8–10]. High spin states up to the $J^\pi = 18^+$ state at 17.961 MeV excitation energy have been observed and successfully described by

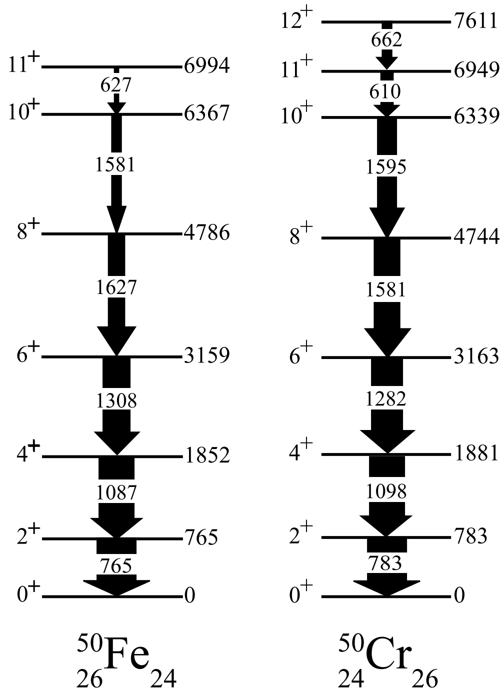

FIG. 2. Yrast states in the $T = 1$ ^{50}Fe and ^{50}Cr mirror nuclei.

spherical shell model calculations. In Fig. 2 only the yrast levels up to the 12^+ state are plotted for comparison with ^{50}Fe. From Fig. 1 the difficulty in identifying the $12^+ \rightarrow 11^+$ transition in ^{50}Fe, which is expected to be a broad line as in ^{50}Cr, is clearly evident. It can be deduced from Fig. 2 that backbending (or upbending) takes place in both mirror partners after spin $J = 8$. From lifetime measurements [9], a sharp decrease of quadrupole collectivity at the 10^+_1 state is observed in ^{50}Cr and the deduced $B(E2)$ values suggest that the ground state band continues in the yrare 10^+_2 and 12^+_2 states [8,9]. This is consistent with full pf shell model calculations [10].

CED studies have contributed decisively to the understanding of backbending as due to an alignment mechanism in odd-mass $f_{7/2}$-shell nuclei: As the nuclear spin increases it becomes energetically favorable to couple pairs of particles to maximum angular momentum. Such an effect will occur both in the spherical and deformed regimes. In the former it will favor high seniority states over low seniority ones. In the latter it will drive the deformed mean field to instability at high rotational frequency. The role of Coulomb is transparent: repulsion is weakest for aligned protons, because the overlap of wave functions is smallest. Hence, we expect a jump in CED at backbend, whose sign will depend on which fluid (neutrons or protons) aligns first. While in odd-mass nuclei, the blocking effect in the odd fluid favors alignment in the even one [3,5], in even-even nuclei the choice is not so evident, but cranked shell model (CSM) calculations indicate that a pair of protons in ^{50}Cr (neutrons in ^{50}Fe) aligns first, and pairs in the other fluid later, at higher rotational frequency [4].

The experimental CED for the $A = 50$ mirrors [$\mathrm{CED}_J = E_J(^{50}\mathrm{Fe}) - E_J(^{50}\mathrm{Cr})$] are displayed in Fig. 3a (solid line). The trend is reproduced but grossly emphasized by shell model calculations in the full pf shell (dashed line). Single particle energies are taken from ^{41}Ca. The interaction is KB3 with Coulomb matrix elements in the harmonic oscillator basis, except for those involving only the $f_{7/2}$ orbits, which are extracted from the CED of the ^{42}Ti-^{42}Ca mirror nuclei. (More on this soon.)

A rapid increase of the CED is observed at $J = 8$ followed by a decrease at $J = 10$. This behavior is consistent with the alignment predictions of CSM calculations [4]. Further insight into the alignment mechanism comes from Fig. 3b. It shows the *difference* of expectation values for the mirrors (^{50}Cr$-^{50}$Fe) of the operator $\mathbf{A} = [(a^+_\pi a^+_\pi)^{J=6}(a_\pi a_\pi)^{J=6}]^0$, which "counts" the number of

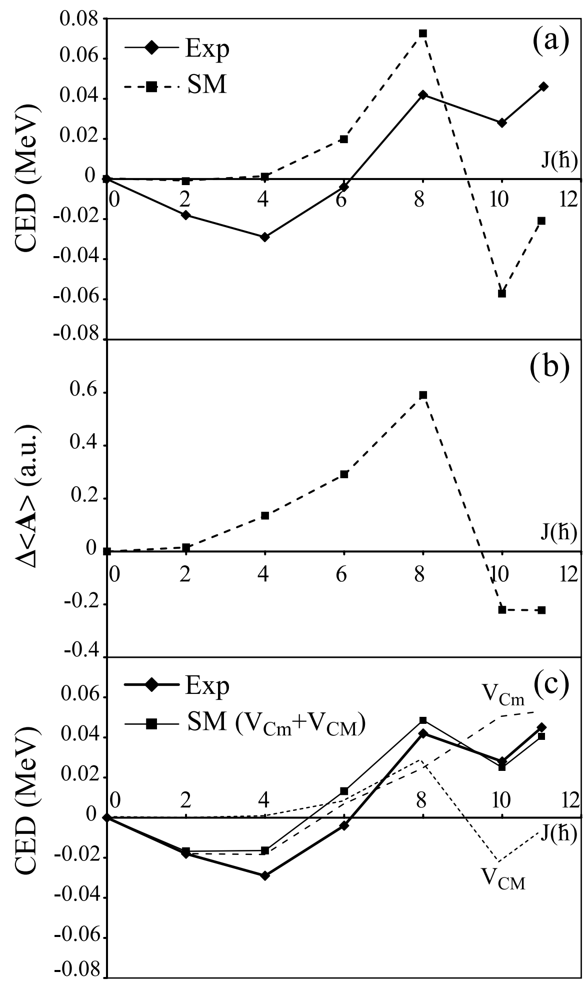

FIG. 3. (a) Experimental CED for the ground state bands in ^{50}Fe and ^{50}Cr (solid line) compared with full *pf* shell model results (dashed line); (b) Difference of expectation values for the mirror nuclei of the operator $\mathbf{A}$; (c) Comparison between the experimental CED and those obtained theoretically taking into account the drift in radii (V_{Cm}) and Coulomb normalizations (V_{CM}).

maximally aligned *proton* pairs in the shell model context [6].

The remarkable agreement between the theoretical curves in Figs. 3a and 3b, however, seems to indicate that the influence of the $J = 6$ pairs in the Coulomb matrix elements is much too strong. Similar problems are encountered in mass $A = 51$ [6] where the effect of the calculated CED agrees nicely with, but overemphasizes, the experimental pattern. In $A = 47, 49$, the observed CED seem to demand an *ad hoc* set of matrix elements, but now it is the $J = 0$ pairs that appear to be too strong [5].

To explain how the present semiquantitative agreement can be improved we shall rely on recent theoretical results [13]. The discussion that follows can be viewed as an introduction to this work, with technical proofs replaced by heuristic arguments. The key point is that the Coulomb matrix elements in the valence space, V_{CM}, represent only the multipole (M) part of the interaction: Another ingredient is necessary for a correct description.

It is associated to $E_{Cm} = 3e^2 Z(Z-1)/5R_\pi$, the monopole ($m$) part of the Coulomb field (R_π is the proton radius). E_{Cm} produces large bulk and displacement energies that cancel approximately in the CED. A small, but significant contribution remains (V_{Cm}), due to changes in yrast states radii. In the pf shell they originate in differences between the f and p orbits. The latter are larger, which leads to the Thomas-Ehrmann shift in ^{41}Sc where the excitation energy of the $p_{3/2}$ level is 200 keV below its analog in ^{41}Ca. The deformed yrast states have large $p_{3/2}$ admixtures, and hence larger radius and smaller Coulomb energies than their aligned counterparts. The effect was identified in Ref. [5] but radii were not mentioned: deformation was taken to be the governing parameter in a Nilsson plus liquid drop context. Within the shell model, changes of radii do not depend necessarily on deformation and can be accounted by the orbital occupancies. This is done in detail in Ref. [13], where it is shown that: (i) E_{Cm} is basically an exact form of the Coulomb energy; (ii) R_π is approximately the same for both members of a mirror pair; (iii) the contribution to the CED of the radial effect is proportional to $\langle m_{p_{3/2}}\rangle$, the total (neutrons plus protons) $p_{3/2}$ occupancy. Note that this number is the same in both mirrors: what makes the difference is Z. In other words: an estimate of the $p_{3/2} - f_{7/2}$ difference in radii can be made through the Thomas-Ehrmann shift, but the radial effect *is not* accounted for by the use of the single particle energies in $A = 41$. The monopole contribution is then $V_{CmJ} = a_m \langle m_{p_{3/2}}\rangle_J$.

The choice of the mulipole contribution V_{CM} raises a problem because the empirical (effective) $f_{7/2}$ matrix elements extracted from $A = 42(V^{\rm eff}_{Cf_{7/2}})$ are much too strong—as seen in Fig. 3a—and very different from those obtained in the harmonic oscillator (ho) basis ($V^{\rm ho}_{Cf_{7/2}}$) [5]. The phenomenological way out proposed in Ref. [13] is to replace the effective interaction in the full pf shell by $V_{CM} = V^{\rm eff}_{Cpf} \approx b_M V^{\rm eff}_{Cf_{7/2}}$ and treat b_M as an adjustable parameter.

The CED can be now written as

$$\begin{aligned}\mathrm{CED}(J) &\equiv V_{Cm}(J) + V_{CM}(J)\\ &= a_m\langle m_{p_{3/2}}\rangle(J) + b_M\Delta\langle V^{\rm eff}_{Cf_{7/2}}\rangle(J)\,. \quad (1)\end{aligned}$$

The V_{Cm} and V_{CM} contributions for $a_m = 0.2$ and $b_M = 0.4$ are shown separately in Fig. 3c. When added, the final CED pattern is quite close to the observed one. In Ref. [13], a single choice $a_m = 0.15$ and $b_M = 0.6$ leads to a consistent description of the CED for the four mirror pairs hitherto observed in the $f_{7/2}$ region. For the present case the agreement is not as good as in Fig. 3c, but still quite acceptable. It is important to note that V_{Cm} can be estimated with good accuracy, while V_{CM} is subject to numerous uncertainties. In particular, the parameter-free choice $V_{CM} = V^{\rm ho}_{Cpf}$ gives a sensible result for $A = 50$. Some renormalization is no doubt in order, though it is unlikely to be as blunt as the one suggested by the $A = 42$ CED. At any rate, the uncertainties in V_{CM} can in no way hide the valuable information about radial behavior uncovered by the monopole term.

In conclusion, the yrast band in the $N = Z - 2$ nucleus ^{50}Fe up to $J^\pi = 11^+$ has been observed for the first time. By comparing with the analog states in its mirror nucleus ^{50}Cr, the obtained CED confirm the alignment mechanism at the origin of the backbending in both $T = 1$ nuclei. In addition, it appears that, in spite of uncertainties in $V^{\rm eff}_{Cpf}$, the CED also offer the possibility of measuring differences in radii along the yrast band, thus bringing to sharper focus the role of the Coulomb interaction as a probe of nuclear structure.

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