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Exergoeconomic analysis of potential magnet configurations of a magnetic refrigerator operating at 4.2 K

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ABSTRACT: This article compares the exergetic cost of cooling of an Adiabatic Demagnetization Refrigerator (ADR) providing 1 W of refrigeration at 4.2 K, with two different magnetic field sources: a Nb_3Sn superconducting (SC) magnet and a NdFeB permanent magnet (PM) Halbach cylinder. The total cost of the system is assumed to be comprised of two components: the cost of the magnetocaloric material (MCM), which is a function of the total volume of the MCM, and the cost of the magnetic system, which depends on the MCM volume and the peak magnetic field. The exergetic cost of cooling for different values of mass (volume) of MCM and hot source temperatures are shown in the article, assuming a specific cost of the SC wire of 890$/kg, 3500$/kg for the MCM, and 100$/kg for the PM. The SC appear to be the most cost-effective solution for the system. However, if large temperatures spans are required between the hot source and the cold source PMs emerge as a better option.

KEYWORDS: Cryogenics; Cryogenics and thermal models

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Contents

1 Introduction

The magnetocaloric effect (or MCE) [1], is a physical phenomenon that occurs in certain materials, which under the exposure of a magnetic field suffer a significant change in entropy. MCE was first observed by Warburg in 1881 [2]. In 1933, Giauque and MacDougall [3] surpass the 1 K barrier achieving a temperature of 250 mK with a magnetic refrigerator (MR). Since then, magnetic refrigeration has been employed to provide cooling over a wide temperature range.

For low temperatures applications, ranging from a few millikelvin to a few kelvin, Adiabatic Demagnetization Refrigerators (ADRs) are used. The refrigeration process followed by an ADR is analogous to a Carnot cycle. The thermodynamics of these devices are described in [4]. For higher temperatures, from hydrogen liquefaction to room temperature refrigerators, magnetic refrigerators have combined the refrigerant material and regenerator material into one device which uses a regenerative cycle, such as Ericsson cycle, to provide cooling. These refrigerators are denominated Active Magnetic Refrigerators (AMRs) [5].

As compared to conventional vapor compression refrigeration, magnetic refrigeration is simple, safe, quiet, compact, and has a high cooling efficiency. Their efficiency superiority is especially notable below 20 K, as traditional gas refrigerators cease to operate efficiently, since the specific heat of the regenerator materials drops off rapidly at these temperatures. This phenomenon can be indirectly appreciated by the low Carnot efficiency reported by these devices [6]. However, higher costs are still the main drawback for this technology, especially due to the need of rare earth materials for the refrigerant material and the magnetic field source.

In this paper the basic principles of the MCE effect and ADR cycles will be reviewed. Afterwards, the main components of an MR will be examined, with special dedication to two possible magnetic field sources: superconducting and permanent magnets. Previous work has been conducted to compare the performance of MRs using both technologies, focusing only ambient temperature applications [7]. In this case, the cost needed for each magnetic configuration will be discussed with the objective of developing an exergoeconomic model for a 1 W ADR type magnetic refrigerator operating at 4.2 K. Such refrigerator could be used for an MRI machine, or scaled in power for other applications that make use of regenerative cryocoolers, with less than 50 W of cooling power, e.g.: for low temperature electronics, or certain particle accelerators applications [6].

2 The magnetocaloric effect

The entropy of a magnetocaloric material (MCM) can be divided in three components [8], the magnetic entropy S_{m}, the entropy of the lattice S_{l}, and the electronic entropy of the material's free electrons:

$$S_T\left(B,T\right) = S_{\mathrm{m}}\left(B,T\right) + S_{\mathrm{l}}\left(T\right) + S_{\mathrm{e}}\left(T\right) \tag{2.1}$$

where the lattice and electronic entropy depend on the material temperature, and the magnetic entropy is dependent on both the magnetic field and the temperature.

If an external magnetic field is applied adiabatically to a MCM, the magnetic moments will tend to align with the field, thereby decreasing the magnetic entropy of the material while maintaining the value of S_T. To compensate for the reduction in the magnetic entropy, lattice, and electronic entropy must increase, which causes an increase in the temperature of the sample. If the magnetic field is withdrawn, the process reverts, the magnetic moments will return to their original alignment capturing energy from the lattice and electronic system, thus reducing the temperature to its original value.

2.1 Thermodynamics of the MCE

A thermodynamic material can exchange energy with an external system through heat and work interactions, which can be expressed as a differential energy balance:

$$\mathrm{d}U = T\,\mathrm{d}S + \mathrm{d}W. \tag{2.2}$$

Work interactions can be expressed more specifically if the energy is exchange in terms of mechanical, chemical or magnetic work:

$$\mathrm{d}W = -P\,\mathrm{d}V + \sum \mu_j\,\mathrm{d}N_j + \mu_0 V_{\mathrm{m}} H\,\mathrm{d}M. \tag{2.3}$$

For a magnetic refrigeration system, where the volume is not modified, i.e. $\mathrm{d}V = 0$, and there is no exchange of chemical energy, eq. (2.2) is expressed as:

$$\mathrm{d}U = T\,\mathrm{d}S + \mu_0 V_\mathrm{m} H\,\mathrm{d}M. \tag{2.4}$$

The total specific entropy change of the system can be represented as:

$$\mathrm{d}s = \left(\frac{\partial s}{\partial T}\right)_H \mathrm{d}T + \left(\frac{\partial s}{\partial H}\right)_T \mathrm{d}H. \tag{2.5}$$

Using the definition of specific heat, $C_{p,H} = T\left(\frac{\partial s}{\partial T}\right)_H$, and the Maxwell relation $\left(\frac{\partial s}{\partial H}\right)_T = \mu_0\left(\frac{\partial m}{\partial T}\right)_{p,H}$, eq. (2.5) can be expressed as:

$$\mathrm{d}s = \frac{C_{p,H}}{T}\,\mathrm{d}T + \mu_0\left(\frac{\partial m}{\partial T}\right)_{p,H} \mathrm{d}H. \tag{2.6}$$

Under the condition that $\mathrm{d}s = 0$, the following expression can be derived:

$$\Delta T_\mathrm{ad} = -\mu_0 \int_{H_\mathrm{I}}^{H_\mathrm{F}} \frac{T}{C_{p,H}}\left(\frac{\partial m}{\partial T}\right)_{p,H} \mathrm{d}H. \tag{2.7}$$

Where H_F and H_I are the final and initial magnetic fields. The expression is denominated as adiabatic temperature change and is the reversible change of temperature that a magnetocaloric material undergoes in an adiabatic process under certain magnetization conditions.

When the MCM undergoes an isothermal process ($\mathrm{d}T = 0$), eq. (2.5) yields:

$$\Delta S_\mathrm{M} = \mu_0 \int_{H_\mathrm{I}}^{H_\mathrm{F}} \left(\frac{\partial m}{\partial T}\right)_{p,H} \mathrm{d}H. \tag{2.8}$$

In this case, the change in entropy is equal to the magnetic entropy change. Both eqs. (2.7) and (2.8) are used to characterize the magnetocaloric effect of certain material. It can be derived from these expressions that the maximum value appears when a significant change of magnetization occurs. This is the reason why magnetocaloric materials are often used near a phase transition, in order to maximize the heat extraction.

To be able to compare among different materials a variable denominated refrigerant capacity (RC) or relative cooling power (RCP) is commonly used, which is defined as:

$$\mathrm{RC}\,(H) = \int_{T_\mathrm{cold}}^{T_\mathrm{hot}} \Delta S_\mathrm{m}\,(T, H)\,\mathrm{d}T. \tag{2.9}$$

Where T_cold and T_hot are the temperature of the cold and hot reservoirs. The advantages of using RC over other parameters is discussed in [9].

2.2 Carnot cycle (ADRs)

Carnot cycles consist of four processes: two adiabatic and two isothermal processes as illustrated in the T-S diagram of figure 1 (left). Other cycles are employed in MR, as Ericsson cycles figure 1 (right), although they won't be explored in this article.

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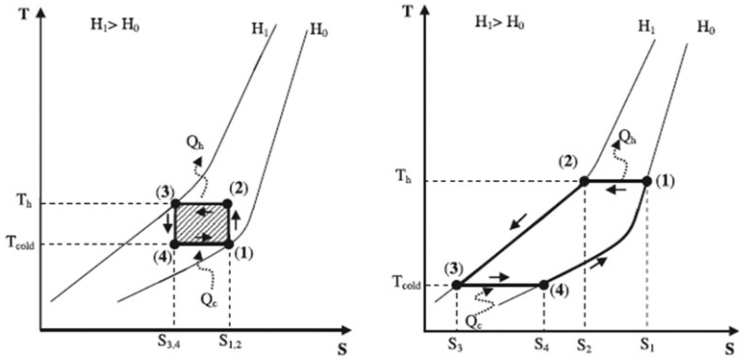

Figure 1. T-S diagram of an MR Carnot cycle (left), and T-S diagram of an Ericsson cycle with regeneration (right). Reprinted from [10].

The first step in the Carnot process (1–2) is an adiabatic magnetization where an external field is applied. The entropy remains constant during the magnetization process. The second step (2–3) is an isothermal magnetization where the heat produced is rejected to the hot source. The third step is an adiabatic demagnetization process lowering the temperature of the MC material. Finally, in the last step, the sample is demagnetized isothermally absorbing heat from the cold source. The area (1–2–3–4) represents the work done during the process, and is equal to:

$$w = \oint_{3}^{2} T\,\mathrm{d}S - \oint_{4}^{1} T\,\mathrm{d}S = T_\mathrm{h}\,(S_2 - S_3) - T_\mathrm{c}(S_1 - S_4). \tag{2.10}$$

Where the first term of the right-hand side of the equation is the heat rejected to the hot source and the second term is the heat absorbed, i.e. the cooling load of the refrigerator:

$$q_\mathrm{c} = \oint_{4}^{1} T\,\mathrm{d}S = T_\mathrm{c}\,(S_1 - S_4)\,. \tag{2.11}$$

The cooling power of the cycle is proportional to the frequency:

$$P = f * q_\mathrm{c} = f * T_\mathrm{c} * \Delta S_\mathrm{c}. \tag{2.12}$$

The maximization of the cycle frequency is a key parameter in the design of a magnetic refrigerator; however, it is limited by the thermal losses produced in the heat exchange process between the refrigerant, and the hot and cold sources. Therefore, the minimization of thermal losses is essential for optimizing the refrigerator. Because of the reversibility of the Carnot cycle, the COP of the

$$\mathrm{COP} = \frac{T_\mathrm{c}}{T_\mathrm{h} - T_\mathrm{c}}. \tag{2.13}$$

In refrigerators, it is useful to define the exergetic cooling power, which is the work equivalent value of the heat flow to the cold source:

$$\mathrm{Ex}_\mathrm{c} = q_\mathrm{c}\left(\frac{T_\mathrm{h}}{T_\mathrm{c}} - 1\right). \tag{2.14}$$

It shows that if the cooling power or the temperature difference between reservoirs is negligible the useful refrigeration is zero in either case.

3 Components of magnetic refrigerator

3.1 Magnetic refrigerant

The MC material is an essential component on the design of a magnetic refrigerator, and two types can be distinguished regarding the order of the phase transition between the ferromagnetic and paramagnetic states near their Curie temperature: first order magnetocaloric materials (FOMT), and second order magnetocaloric materials (SOMT). The former undergoes a discontinuous change in magnetization with temperatures, while the latter undergoes a continuous change. Comprehensive reviews regarding the different MCM exist within the literature [11] from low to ambient temperatures applications.

Some authors have provided a practical set of selection rules for picking a magnetic refrigerant depending on the application [12]. Some of these rules are: the selection of a suitable Curie temperature, intensity of the magnetocaloric effect, high electrical resistivity (to prevent eddy currents), or good manufacturing and corrosion properties.

3.2 Magnetic field sources

The magnetic field is a crucial part of the magnetic refrigerator. There are two potential sources: electromagnets, or permanent magnet assemblies. Among the first, two types are recognized: superconducting or copper electromagnets. The design of the magnetic field source is key since it is usually the most expensive component, representing in some cases up to 85–90% of the cost [13], although it will vary depending on the application. In [12] a comprehensive review of the different magnetic field sources and their characteristics for MR is found. In the following section, superconducting magnets and a cylindrical Halbach array using permanent magnets will be examined, as both are the most common options for MRs. Superconducting magnets are capable of providing high and stable magnetic fields with low space requirement, PMs provide smaller fields although it is not necessary to cool them down to cryogenic temperatures (figure 2).

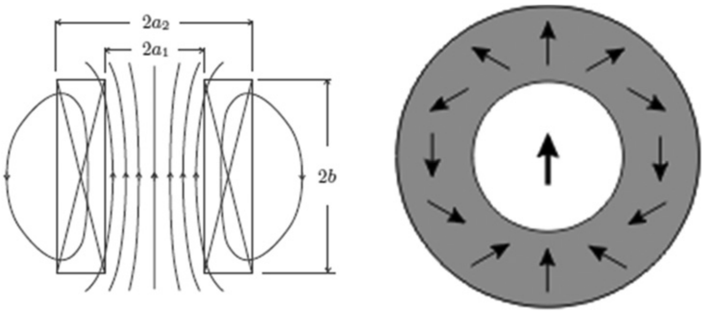

Figure 2. Two types of magnetic field sources: solenoid, a type of electromagnet (left) and a Halbach cylinder made of permanent magnets (right).

3.2.1 Electromagnets: superconducting magnets

In MR applications, superconducting magnets have been utilized since the early beginning of the field. It have been used in refrigerators from very low temperatures to room temperature refrigeration, some of the developed prototype devices can be found in [11].

A typical superconducting system is composed of three main parts: a superconducting coil, a cryogenic system, and a power conditioning system. The topology of the superconducting coil in magnetic refrigerators is typically a solenoid, where the aperture of the magnet is a cylindrical volume where the MCM is placed. There is plenty of information in the literature concerning the design of superconducting solenoids [14].

The main variable when designing a magnet for MR is the magnetic field, which is not homogenous in all the volume, neither in magnitude nor direction. In a SC solenoid, where there is no ferromagnetic material, the magnetic field at the center is given by the following equation:

$$B_z(0,0) = \mu_0 \lambda J a_1 F(\alpha, \beta). \tag{3.1}$$

Where μ_0 is the magnetic permeability in the vacuum, λ the field factor, J the current density, a_1 the internal radius, and $F(\alpha,\beta)$ the field factor which depends on the geometric parameters [14] $(\alpha = \frac{2a_2}{2a_1})$ and $(\beta = \frac{2b}{2a_1})$.

3.2.2 Permanent magnets: Halbach cylinder

A permanent material is a magnetic material which remain magnetized after the withdrawal of an external magnetic field. Permanent magnets materials are usually divided into: ceramics materials, rare-earth materials, Al-Ni-Co materials and polymer bonded materials. A review regarding their composition and critical properties can be found in [15].

A way to classify a permanent magnet array is to consider the figure of merit, M^*, which according to [16] is equal to:

$$M^* = \frac{\int_{V_\text{field}} ||\mu_0 H||^2 \mathrm{d}V}{\int_{V_\text{mag}} ||B_\text{rem}||^2 \mathrm{d}V}. \tag{3.2}$$

Where V_field is the volume where the magnetic field ($\mu_0 H$) is created, V_mag is the volume of the permanent magnets and B_rem the remanence, the magnetization left after the removal of the external magnetic field. If it is assumed that the magnetic field is constant in all the volume, as well as the remanence, eq. (3.2) gives:

$$M^* = \frac{\left(\frac{\mu_0 H}{B_\text{rem}}\right)^2 V_\text{field}}{V_\text{mag}}. \tag{3.3}$$

Which if V_field is substituted by the mass of the MCM divided by its mass density, and one minus the porosity of the regenerator, and also substituting the volume of the magnet by its mass divided by the density, the following expression can be derived:

$$m_\text{magnet} = \left(\frac{\mu_0 H}{B_\text{rem}}\right)^2 \frac{m_\text{MCM} \rho_\text{mag}}{(1-\varepsilon)\, \rho_\text{MCM} M^*}. \tag{3.4}$$

In which the mass of the permanent magnets is related to its magnetic and mechanical properties, and to the mechanical properties of the MCM material.

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4 Exergoeconomic model

The term exergoeconomics is used to describe the combination of an economic and exergy analysis [17]. In an exergoeconomic balance each exergy stream is associated with a cost. Using this methodology, the cost balance of a refrigerator would be:

$$C_{\mathrm{c}} = C_{\mathrm{capex}} + C_{\mathrm{op}}. \tag{4.1}$$

Where C_{c} is the cost rate of cooling, C_{capex} is the cost rate of capital, the equipment, and C_{op} is the cost rate of operation of the device during its lifetime, which includes operation and maintenance costs. In a detailed analysis, operating costs should be included. However, as they represent a small fraction of the total cost, independently of the solution proposed, they will be neglected, e.g.: a 4.2 K, 1 W MR with a 50% Carnot efficiency [12], will have an equivalent 150 W electric consumption, which with an electricity cost of 0.1$/kWh yields an operating cost of 131$ per year (less than 3% of the capital costs as will be shown later).

Capital costs should be amortized for the expected life of the device in order to transform them into a cost rate. For that purpose, it is used the capital recovery factor (CRF):

$$C_{\mathrm{capex}} = \mathrm{CRF} * Z. \tag{4.2}$$

Where Z are the absolute capital expenses, and CRF is given by:

$$\mathrm{CRF} = \frac{i(1+i)^n}{(1+i)^n - 1}. \tag{4.3}$$

If c_{c} is defined as the cost per unit cooling, such as C_{c} is given by the product of c_{c} and the cooling exergy, E_{c}. Eq. (3.4), neglecting operating costs, would be expressed:

$$c_{\mathrm{c}}\mathrm{Ex}_{\mathrm{c}} = \mathrm{CRF} * Z. \tag{4.4}$$

In the case of a magnetic refrigerator, the capital costs Z, are mainly due the cost of the magnet and the MCM. Other costs are ignored. Hence, the cost per unit of exergetic cooling is:

$$c_{\mathrm{c}} = \mathrm{CRF} * \frac{Z_{\mathrm{magnet}} + Z_{\mathrm{MCM}}}{\mathrm{Ex}_{\mathrm{c}}}. \tag{4.5}$$

The cost of the refrigerant material is easily determined as it can be defined as the product of the cost per unit volume c_{MCM} and the total volume of material used, V_{MCM}. Establishing the cold source temperature, the cooling power needed, the lower magnetic field (typically 0 T), and the operating frequency, a relation between the mass of MCM needed, the hot source temperature and the value of the higher magnetic field, for an ADR, can be obtained with eq. (2.12), and the isentropic equality in the process 3–4. If one of the three variables is fixed, the others can be immediately obtained. The cost of the magnetic field source is related to the mass of the MCM, and the value of the higher magnetic field as will be shown in the following sections.

4.1 Superconducting solenoid cost

The capital cost of a superconducting magnet can be related to the superconducting material mass used in the magnet. To compute the mass, the following assumption will be made: that the length

(2b) of the magnet is much greater than the internal radius. This is particularly true for a magnet of these characteristics, as with this configuration the magnetic field would be more homogenous in the internal volume, where the magnetocaloric refrigerant will be placed. With this assumption, the magnetic field at the center is:

$$B_z\,(0,0) = \frac{\mu_0 NI}{2a_1\beta}. \tag{4.6}$$

Full derivation of this terms can be found in [14]. It is also known that the current density should be equal to the total ampere-turns divided by the cross section of the magnet:

$$\lambda J = \frac{NI}{a_1^2\beta\,(\alpha - 1)}. \tag{4.7}$$

Therefore, being $V_{\text{MCM}} = 2\pi a_1^3\beta$ the volume of the magnetocaloric material, and $V_{\text{Magnet}} = 2\pi a_1^3(\alpha^2 - 1)\beta$, the following expression can be derived:

$$B_z\,(0,0) = \frac{\mu_0 J V_{\text{mag}} a_1}{V_{\text{MCM}}\,(\alpha + 1)} = \frac{\mu_0 J V_{\text{mag}}\rho_{\text{MCM}} a_1}{V_{\text{MCM}}\rho_{\text{mag}}\,(\alpha + 1)}. \tag{4.8}$$

4.2 Permanent magnet cost

Eq. (3.4) gives a relation between the magnet mass and the mass of MCM, therefore if the unit cost per kg of the PM is known, it would be possible to establish the cost. However, M^* is not yet defined. For a Halbach cylinder of infinite length it can be shown through the relation of the field in the bore, $\mu_0 H = B_{\text{rem}}\ \ln(\frac{r_o}{r_i})$, that the figure of merit M^* is [18]:

$$M^* = \frac{\left(\frac{\mu_0 H}{B_{\text{rem}}}\right)^2}{e^{2\frac{\mu_0 H}{B_{\text{rem}}}} - 1}. \tag{4.9}$$

For NdFeB magnets, with a remanence of 1.2 T, there is an optimum value at $\frac{\mu_0 H}{B_{\text{rem}}} \approx 0.8$, which yields a figure of merit $M^* \approx 0.162$. Although this equation could yield values over 3 T for the Halbach cylinder, it would be limited to that value due to practical motives.

5 Results

In this section, the costs of two identical magnetic refrigerators in terms of cooling, one with a superconducting magnet and the other with a permanent magnet, will be compared in terms of cost per unit of exergetic cooling. Both systems will have a refrigeration power of 1 W at 4.2 K, operating at a frequency of 0.05 Hz, with a regenerator porosity of 0.4, and will use GGG as magnetic refrigerant. GGG properties has been discussed in [19]. Figure 3 (left) shows the entropy dependence of GGG on temperature and magnetic field. The assumed unit cost of GGG is 3500$/kg, provided by American Elements [20].

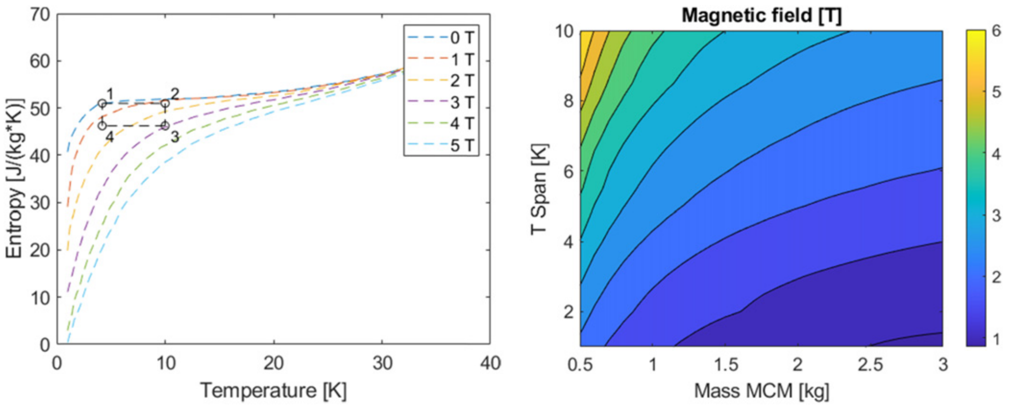

Figure 3. Entropy of GGG as a function of temperature [K] and magnetic field [B] (left), and magnetic field dependence on mass and hot source temperature to provide 1 W at 4.2 K of cooling (right).

The hot source temperature, and the mass of the magnetocaloric material will be the iterative variables, which will iterate between 1–10 K and 0.5–3 kg., respectively. Figure 3 (right) shows the peak magnetic field in the regenerator as a function of the mass of the MCM, and the hot source temperature. Although, the direction of the magnetic field created by the superconducting magnet, and the Halbach cylinder, are not in the same plane, it has been assumed that the demagnetization factors are equal in both directions and the average field in the regenerator is equal to the peak field.

The superconducting magnet is considered to be operating at the temperature of the hot source, and the cost of refrigeration has not been included. In this case, Nb_3Sn technology has been considered [21], for which a fitting function [22] has been used to extrapolate the current density values to other temperatures and magnetic fields. In this case, NbTi is not suitable due to its reduced range of operating temperatures, however HTS could be of interest for the opposite reason. It is assumed that the filling factor λ, is equal to 0.8, and the working point of the magnet is 0.75 [14]. A unit price of 890$/kg has been used for the SC magnet, provided by the manufacturer [21]. For the permanent magnet a price of 100$/kg [7] is used. Since both devices are expected to have a similar lifetime, the capital recovery factor has been omitted.

5.1 Refrigerator cost at 4.2 K

Having computed the maximum field in the regenerator the derivation of the costs of the magnetic systems is straightforward, using eq. (4.7) for the superconducting magnet, and eqs. (3.2) and (4.8) for the permanent magnet configuration. Figure 4 shows that the minimum value cost per unit of heat transfer is obtained with the use of a superconducting magnet, with a MCM mass of around 0.6 kg and a hot source temperature of 8 K. If this configuration is compared against figure 3, it is observed that the optimum magnetic field is in the range of 3 T, much lower than the maximum magnetic field achievable by a Nb_3Sn superconducting magnet, and used in previous refrigerators [23]. It is also seen that the cost rapidly increases as the hot source temperature increases. This is due to the deterioration of the current density with temperature.

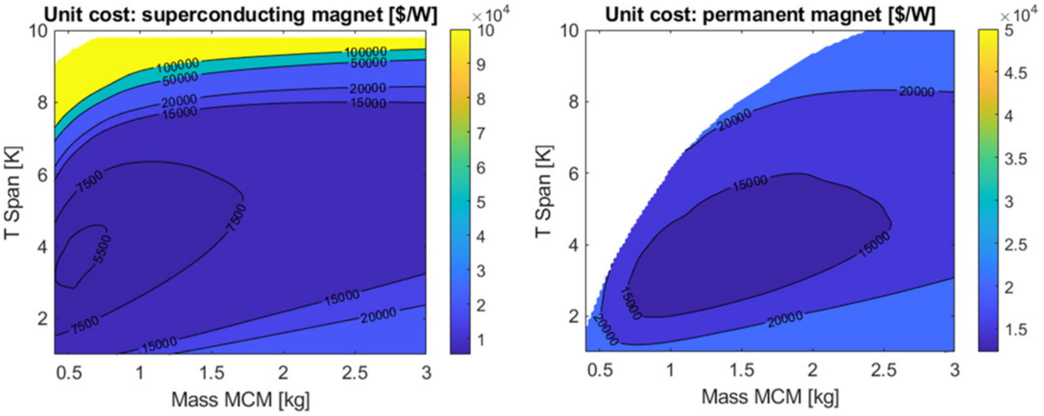

Figure 4. Cost per unit of heat transfer for a magnetic refrigerator providing 1 W at 4.2 K, with a superconducting magnet (left) and a permanent magnet (right). The white area, in both graphs, comes from the impossibility of achieving the required magnetic fields in the specific operating conditions.

On the other hand, the permanent magnet configuration shows a different performance. As expected, the minimum values of cost appear with higher mass of the MCM than in the previous configuration, this implies lower magnetic fields on higher volumes. It is also noteworthy to observe in the Halbach array configuration, the cost dependence with the hot source temperature. In this case, the increase with temperature is much slower than with a superconducting magnet, as the magnetic properties of the permanent magnet do not depend on temperature. For hot source temperatures over 12–13 K, the permanent magnet configuration appears to be more cost-efficient if capable of providing the required magnetic field.

6 Conclusions

The unit cost of a 1 W ADR type magnetic refrigerator operating at 4.2 K was determined over different hot sources temperatures, and values of MCM mass, for two possible magnetic sources: a Nb_3Sn superconducting magnet, and a NdFeB permanent magnet in a Halbach array. Assuming a cost of 890$/kg for the SC, and 100$/kg price for the NdFeB, it was shown that the most cost-effective solution was using a superconducting solenoid operating with an optimum magnetic field of 3 T. However, if the temperature span, between the cold source and hot source increases over 8 K, the permanent magnet configuration becomes more cost-effective. This creates the necessity (2.1) to further study the use of PMs in MRs with large temperature differences, spanning from LN2 (77 K), to LH2 (20 K) to LHe (4.2 K). (2) to explore the possibility of using HTS superconducting materials over large temperatures spans for MRs. Likewise, further research is needed in the magnetic optimization of other types of thermodynamic cycles more appropriate to temperatures over 20 K, such as AMRs.

Acknowledgments

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